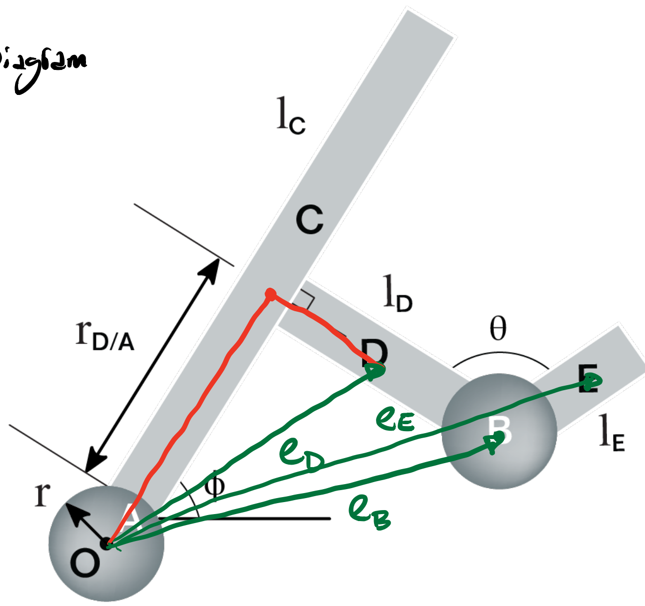
An engineer puts together a form study prototype of a robotic arm to show a group of stakeholders. Specifically they want to know about its radius of gyration. Unfortunately, he forgot what material he used. If the mass moment of inertia of the arm is $I = 15.2\ kgm^2$ about point O, calculate the radius of gyration. Each component is a plate with thickness $t = 5\ mm$. Assume the plates are rigidly attached to one another.
Plate A is identical to plate B, and has a radius $r = 2w$.
Plates C, D and E have the same width $w = 15\ cm$.
Plate C has a length $l_C = 1.1\ m$, and is angled at $\phi = 30\ deg$ with the horizontal.
Plate D is attached perpendicular to plate C at a distance $r_{D/A} = 0.55\ m$ from plate A, and has a length $l_D = 0.3\ m$.
Plate E has a length $l_E = 0.21\ m$, and is angled $\theta = 105\ deg$ away from plate D.

① Diagram



$$K = \sqrt{\frac{I}{m}}$$

Disk A: $$m = \rho V_A = \rho \pi r^2 t = \rho \pi (0.3m)^2 (0.005m)$$

$$I_{zz} = \frac{1}{2} m r^2 = \frac{1}{2} \rho \pi (0.3)^2 (0.005)\ kgm^2$$

Plate C: $m = e V_C = e(1.1\,m \times 0.15\,m \times 0.005\,m) = 0.000825e\,[kg]$

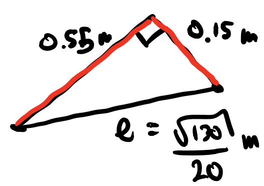
$$I_{zz} = \frac{1}{12} m (a^2 + b^2) = \frac{1}{12}(0.000825e\ kg)\left((1.1\,m)^2 + (0.15\,m)^2\right)$$

$$I_{OC} = I_{zz} + m\ell^2 = \frac{1}{12}(0.000825e\ kg)\left((1.1\,m)^2 + (0.15\,m)^2\right) + 0.000825e\ kg\,(0.55\,m)^2$$

Plate D: $m = eV = e(0.3\,m \times 0.15\,m \times 0.005\,m) = 0.000225e\ kg$

$$I_{zz} = \frac{1}{12} m (a^2 + b^2) = \frac{1}{12}(0.000225e\ kg)\left((0.3\,m)^2 + (0.15\,m)^2\right)$$

$$I_{OD} = I_{zz} + m\ell^2 = \frac{1}{12}(0.000225e\ kg)\left((0.3\,m)^2 + (0.15\,m)^2\right) + 0.000225e\ kg\left(\frac{13}{40}\,m^2\right)$$



Disk B: $m = e\pi(0.3\,m)^2(0.005\,m)$

$$I_{zz} = \frac{1}{2} e\pi(0.3)^4(0.005)\ kg\,m^2$$

$$I_{OB} = I_{zz} + m\ell^2 = \frac{1}{2} e\pi(0.3\,m)^4(0.005\,m) + e\pi(0.3\,m)^2(0.005\,m)(0.626\,m)^2$$

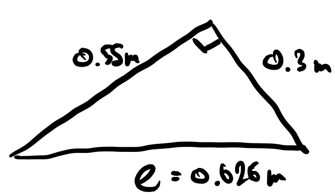
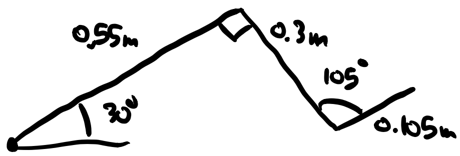


Plate E: $m = e V_E = e(0.21\,m \times 0.15\,m \times 0.005\,m) = 0.0001575e\ kg$

$$I_{zz} = \frac{1}{12}(0.0001575e)\left((0.21\,m)^2 + (0.15\,m)^2\right)$$

$$I_{OE} = I_{zz} + m\ell^2 = \frac{1}{12}(0.0001575\rho)\left((0.21m)^2 + (0.15m)^2\right) + 0.0001575\rho\,(0.8454\ m^2)$$



$$x = 0.55\cos 30^\circ + 0.3\cos 60^\circ + 0.105\cos 15^\circ = 0.7277m$$

$$y = 0.55\sin 30^\circ + 0.3\sin 60^\circ + 0.105\sin 15^\circ = 0.5619m$$

$$\ell^2 = 0.8454\ m^2$$

$$I_{TOT} = I_A + I_{OB} + I_{OC} + I_{OD} + I_{OE} = 0.0012\rho\ [kg\,m^2]$$

$$\rho = 1.2401 \times 10^4\ \frac{kg}{m^3}$$

$$m = V\rho = 50.04\ kg$$

$$K = \sqrt{\frac{I_{TOT}}{m}} = \sqrt{\frac{15.2\,kg\,m^2}{50.04\,kg}} = \boxed{0.55\ m = K}$$