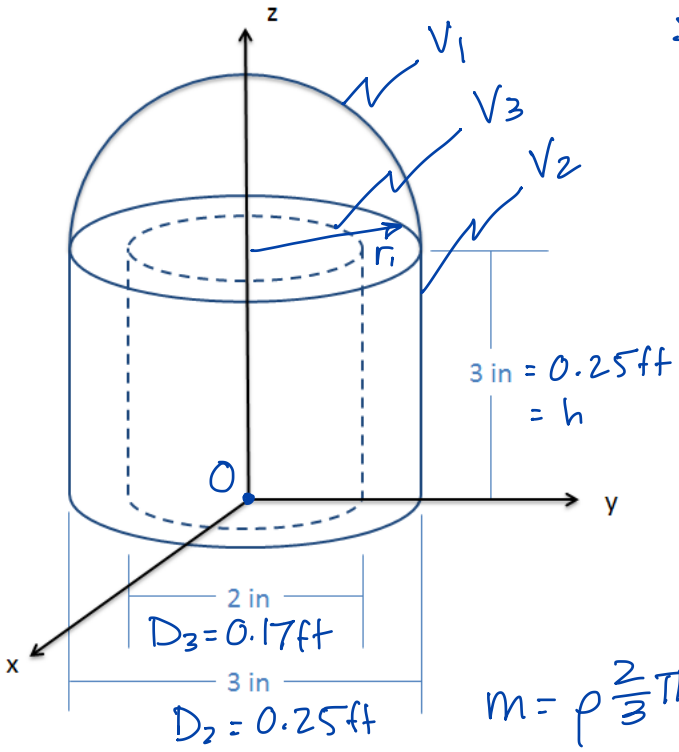
The shape shown below consists of a solid semicircular hemisphere on top of a hollow cylinder. The object has a constant density, and a mass of 5 slugs. Based on the dimensions below, determine the mass moment of inertia about (a) the z-axis and (b) the y-axis.

MMOI units: $kg\text{-}m^2$, or $slug\text{-}ft^2$



Find density, to find masses

$m = m_1 + m_2 - m_3$

$m_1 = \rho V_1 = \rho \frac{2}{3}\pi r_1^3$ $\quad r_1 = r_2 = \frac{D_2}{2} = 0.125\,ft$

$m_2 = \rho V_2 = \rho \pi r_2^2 h$

$m_3 = \rho V_3 = \rho \pi r_3^2 h$ $\quad r_3 = \frac{D_3}{2} = 0.083\,ft$

$m = \rho \frac{2}{3}\pi r_2^3 + \rho \pi r_2^2 h - \rho \pi r_3^2 h$

$\frac{m}{\pi} = \rho\left(\frac{2}{3}r_2^3 + r_2^2 h - r_3^2 h\right)$

$\frac{m}{\pi} = \rho\,(0.003486\,ft^3) \Rightarrow \rho = 456.8\ slug/ft^3$

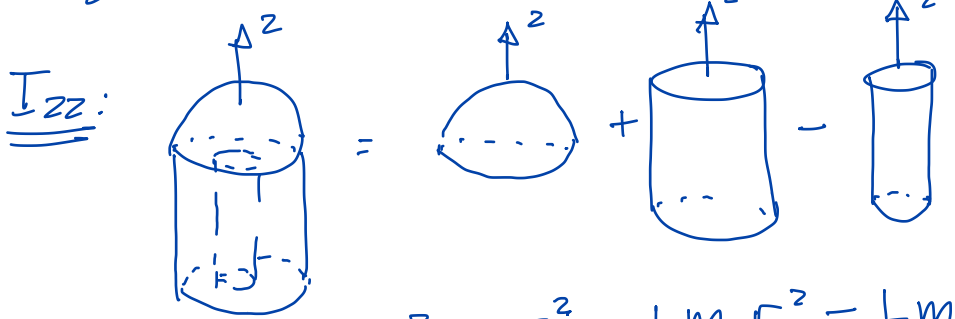
$m_1 = 1.87$ slugs

$m_2 = 5.60$ slugs

$m_3 = 2.47$ slugs.

check: $1.87 + 5.60 - 2.47 = 5$ slugs.

I_{zz}:

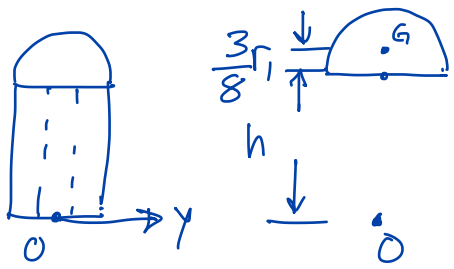


$I_{zz} = \frac{2}{5} m_1 r_1^2 + \frac{1}{2} m_2 r_2^2 - \frac{1}{2} m_3 r_3^2$

r_2^2

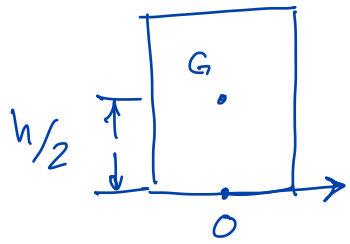
$I_{zz} = 0.0470\ slug\text{-}ft^2$

I_{yy}:



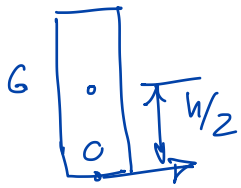
$$I_{yy,G} = 0.259\, m_1 r_2^2 \quad (r_1 = r_2)$$

$$I_{yy,O} = I_{yy,G} + m_1\left(\tfrac{3}{8}r_2 + h\right)^2$$



$$I_{yy,G} = \frac{1}{12} m_2 (3r_2^2 + h^2)$$

$$I_{yy,O} = I_{yy,G} + m_2\left(\frac{h}{2}\right)^2$$



$$I_{yy,G} = \frac{1}{12} m_3 (3r_3^2 + h^2)$$

$$I_{yy,O} = I_{yy,G} + m_3\left(\frac{h}{2}\right)^2$$

$$I_{yy,O} = I_{yy,O} + I_{yy,O} - I_{xy,O}$$

$$\boxed{I_{xy,O} = 0.255 \text{ slug-ft}^2}$$