Problem **20-R-KIN-DK-12**

In this problem, we're asked to determine the radius of gyration of the following structure about point O. So we have two disks, A and B have the same dimensions. And then we have three plates, C, D, and E, have different dimensions, everything is of constant thickness into the page. And, again, disks A and B are of the same dimensions and all the other geometrical properties are given. Now we're given the mass moment of inertia. And we're asked to find the radius of gyration. So since we are given the moment of inertia, we can actually write out the formula for the radius of gyration, which is k radius of gyration is equal to the square roots of the mass moment of inertia divided by the mass. Now we have I, and we are asked to find K, so all we need to determine is the mass. Now, we're, again we're not given a density and we're not given a mass, so we're going to have to solve for that. And we can solve for the mass based on the mass moment of inertia, because mass moment of inertia depends again, on the density or the mass of the material and then geometrical properties which were given. So we're gonna have to find a expressions for the mass moment of inertia from the components of all of these parts of the system, and then combine them combine this equation equated to this mass moment of inertia, and solve for the mass of the structure. Now, since there's many components, we can, each of these components will have a different mass because it has a different geometry. So we can't actually directly solve for mass in that equation, we actually have to solve for the density. And once we solve for the density, then we can go back and solve for mass. So again, we have to solve for the density of the material, which is constant throughout the structure. So we're actually since we're taking in about point O here, we're going to start with a structure that is centered at o, which is the first plate and then we're going to calculate the mass moment of inertia I for each of these five distinct shapes. Remember, the mass moment of inertia about a point is specific to the shape. If the center of mass is away from that point, we also have to use parallel axis. So for example, in this case, this midpoint, we need to shift with parallel axis, but start at disc a disc a, we first need to calculate the mass, and then we can plug that into the equation for the mass moment of inertia. So the mass of disk a is going to be equal to the density of the material times the volume of disk a, and the volume of disk a is going to be equal to the thickness times the area, and the area is just the area of a circle. So pi r squared, so we have the density times pi r squared times thickness t. And so this is going to or you can actually plug in everything except for the density, which is the variable we're solving for. We have rho pi times 0.3 meters to the power of two. Again, this is the radius, then times 0.005 meters, and that's the thickness, okay? And then we can actually solve we can plug that into the formula for eyes at Zed, which is again, the mass moment of inertia. And that equation is one half and r squared for a disk and we can plug in and so we get one half rho pi 0.3 squared 0.005 and this is going to be kilogram kilogram meter squared. All right, so this is for disc a. Now we're going to do the same process for our plate See, sorry, my plate see the center of mass is located away from point O about which we are measuring on the web but which were given the mass moment of inertia. So we need to use parallel axis. So we're going to calculate again, the rotation, the mass moment of inertia about the center of mass. And then we're going to shift it to point O based on that distance with M L squared, l being the distance between this point and between the center of mass and point oh, so let's do that. So for our mass is going to be equal to rho times the volume of C. Now volume of C is base times height times steps because it's a plate. And so we're just going to plug those in 1.1 meters, times 0.15 meters times 0.005 meters. And that's going to get a nuclear cell for everything that 0.000825 row. kilograms, vary the units. Then we again, so for I Zed Zed, which is equal to for a thin plates, or for a plate, 112 A squared plus B squared, where this is the length, and this is the width. So we can plug those in. And, and then again, we have to use parallel axis on so if we plug those in, we get one over 12 M, which is what we had before, times 0.000825. Row, kilogram grams. And now we add the length and the width added squared. So we have 1.1 meters squared plus 0.15 meters squared. Hey, and now we have to use parallel axes to get I about point O of C. And this is going to be equal to eyes Zed plus L squared. And again, this L is the distance between this point oh and the center of mass of points of C. So that's, we can just essentially plug in all the values, and we get the following. We had the morass 0.000825 row, kilograms, times the language is 0.55 meters squared. And if we, and then we're going to, we can simplify this, but I'll simplify it later. So essentially, this is a function of just right here and over here. Alright, now we can move on to the next plate, which is plate D. And it's a bit more intricate to find the distance between Oh, and this center point here on so we'll use some trigonometry, but again, everything else is the same as plate C. So plate D, is going to have mass equal to rho times the volume, which is equal to rho times 0.3 meters, times 0.15 meters times 0.005 meters. Again, this is base times width times height. This is going to be equal to 0.000225 rho kilograms. Okay, and now we can solve for either Zed. Zed Zed is going to be equal to one over 12 and a squared plus b squared, again, length and width of the plate. And that's going to be equal to one over 12 times 0.000225 rho kilograms times a squared, which in this case is 0.3 meters, all squared, plus 0.15 meters all squared. Okay, so that's ies Zed, and now we have to find i O, PR, D. So again, we're going to use parallel axis, and it's going to be equal to eyes and lead plus L squared. This L squared is the distance between this point and midpoint because the center of mass is distributed at the middle of that plate. So we're going to use some trigonometry to solve for that distance. So we have that 0.55 distance to the left, so that's the radius of barsi. And then halfway down that plate D is going to be 0.15 meters, these are both meters. And this is going to be a right angle, because we're given that. So we can use Pythagoras to find this distance here, which is going to be L. So l is going to be equal to the square root of 130 over 20. So now we can plug l in to this equation over here. And we have eyes at Zed. And we have our math over here. So we can essentially solve for IO D with respect to rho. So this is the following equation that we get. Plus 0.000 to five row kilograms, times 13 over 40 meters squared. And again, this whole equation here is in terms of rho. So we've done play D. Now let's move on to disc B. So again, this B uses the same formula as disc a, but now we're offset from this point oh, so we were offset from point O. So we have to add this distance. So let me draw in the distances, so it's, it's clear. So this is going to be L for plate D. And this over here is going to be L four, B, okay. And the triangle I drew before was this triangle over here. This triangle over here, so in red over here, like that. Um, so now we're gonna do, the angles are a bit different. So essentially, we do Pythagoras again. But this time, we're going to go all the way to the end of that D. So let's offer the math first, just like we did before, the mass is going to be identical to the mass of disk, a because they have the same dimensions. So the mass is going to be equal to rho pi times 0.3 meters square times 0.005 meters. Now we can find I Zed Zed. This is equal to the same as the one before, it's the same dimensions. So I'm just going to directly input what we have Or twos. And this is in kilograms meters squared. And now we can find I have about Oh, from this beat. And that is going to be equal to I have that said, plus L squared. And now it's a bit more challenging to find out, because l will be, again, this here will be, so we're just going to travel further down the full distance of LD. So redrawing that triangle, we get the following. So again, we have this right angle over here, we have 0.55 meters. And here we have 0.3 meters instead of the half, which is 0.15 meters. Again, just to be clear, we're assuming that like this bar attaches at the midpoint here, and at the midpoint of B. And again, l, e is from the center of the to the end of E. Okay. So that's where the distances are from. And so now we can find that l that distances with just through Pythagoras, is going to be equal to 0.626 meters. And we can plug it into our equation and solve for iob with respect to rope. So again, this is nearly one half rho pi times 0.3 meters to the for 0.000005 meters, plus, on the second component, which is rho, times pi, times 0.3 meters, everything squared times 0.005 meters times 0.626 meters squared, again, because it's M L squared, so I just took that length squared. And then we were essentially we solve for iob. Now let's move on to plate eat. So plate II, again, it's a bit more complex geometry, because now the distance or l for a parallel axis, start from here and we end up the midpoint of E here, it's going to be L e. And essentially, we use the same concept, there's just a bit more trigonometry involved in this problem. So like always, let's solve for the mass first. So the mass is going to be equal to the density times the volume of E, which is going to be equal to rho times 0.21 meters times 0.0. point one five meters, times 0.005 meters. And this is going to be equal to 0.00015750 kilograms. And so I just essentially, multiplied all these together. And now I can find I Zed Zed. So my Zed Zed is going to be equal. So it's the one for plate on 12th of M, and then a squared plus b squared, where a and b are the length and the width. So we have 0.0001575 rho times 0.21 meters squared plus 0.15 meters squared. And now again, to solve for I about point O from E, we have I have Zed Zed, plus And l squared. So we have a Zed, we just need to find out. So l is a bit more intricate. So I'm just going to draw out all the arms. So we know that this angle here is going to be 30 degrees. This angle here is 105 degrees. And we know that this is the right angle, this is 0.55 meters. And this is 0.3 meters. And the last one is 0.105 meters 105, because we're only getting to the halfway point, right, and this is not the full length of E, we're only getting to the halfway point, or the center of mass. So we can, we're just going to do it in the x and y coordinates. So we're going to find, so we're gonna assume this is zero. And we're going to find the x&y coordinates of this point, and then sum the squares to get that distance over there with Pythagoras. So it's just going to involve some trigonometry. So the x component is going to be equal to 0.55 times cosine of 30 degrees, plus 0.3 times coasts of 60 degrees. And this is if you do Pythagoras, this is essentially the component of this thing he have from this arm here, the x component, and then we have plus 0.105 coasts of 15 degrees, you're going if you do Pythagoras, you see that this angle here is 15 degrees. And so that's our x component. And it's going to be equal to 0.7277 meters, where you can solve for the y component, which is just everything terms of sine, not cosine, so 0.55 sine of 30 degrees, plus 0.3, sine of 60 degrees, plus 0.105, sine of 15 degrees, which is equal to 0.5619 meters, then we take the sum of the squares to find L squared. So l squared is equal to 0.8454. And this is going to be in units of meters squared, because again, it's L squared, l, I just square these two. And because they're x and y components, and took the sum of them. So now I have L squared, I can directly plug it into here. And so for i o e in terms of row with this, which is the density, so we have one over 12 0.0001575 rho times 0.21 meters squared plus 0.15 meters squared. And then we add this new component from parallel axes, which is 0.0001575 rho times L squared, which is 0.8454 meters squared. And we have io e. So now we have all of the components for I which is IO, e, i, o BIODIOC. And IOA, which is just I said, because it's centered, so we can add them all up and equate them to the value of 15.2 kilograms meter square that we have. So I total is equal to IA plus i o, b was i o c, plus IOD plus i o e, and this if we add it up, we're going to get the following Yeah, so that is I total. Now we also know that a total is equal to 15.2. So we can solve for row row is going to be equal to 1.2401 times 10 to the negative 10. Seven to the four kilogram kilograms per meter cubed. So this is the density. Now we have the density, so we can find the mass. So remember the relation between mass volume and density is that the math is equal to the volume times the density. And we have, we now have the density so we can actually solve for the total mass, which is going to be equal to and we also already have the volume, the volume is just adding up all of the of these volumes that we have he, then we have this is the volume or we can just sum up all these masses and Add rho in it, same thing. But, essentially, and I mean, right this clearly, so didn't look like p rho, and we get a mass of 50.04 kilograms. Okay, I skip the plugging in because it takes a lot of space, but if you just plug it in, you get that value. And once you have this mass, then we can finally solve for that radius of gyration. So, k is equal to the square roots of I total divided by the mass, which is going to be equal to if we plug in square root of 15.2 kilogram meter squared divided by 50.04 kilograms equal to 0.55 meters and is equal to the radius of gyration.