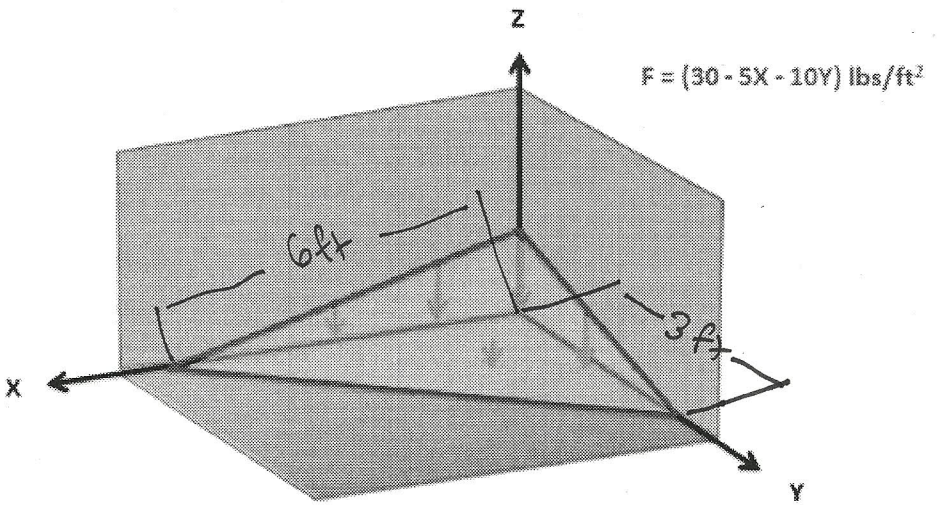
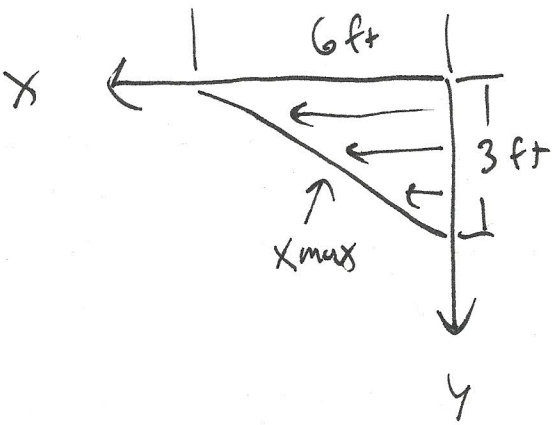
Question 4:

The wind has piled up sand into a corner on a building. The building supervisor is worried about the weight of the sand pushing against the roof of the basement below. The function describing force of the sand pushing down on the surface is given below. Find the magnitude, direction and point of application of the equivalent point load for the distributed force of the sand. Draw the equivalent point load in a diagram.





$y_{min} = 0$

$y_{max} = 3$

$X_{min} = 0$

$X_{max} = 6 - 2Y$

$$F_{eq} = \int_0^3 \left(\int_0^{6-2y} (30 - 5x - 10y)\, dx \right) dy$$

$$F_{eq} = \int_0^3 \left(\Big|_0^{6-2y} 30x - \frac{5}{2}x^2 - 10yx \right) dy$$

$$F_{eq} = \int_0^3 \left(30(6-2y) - \frac{5}{2}(6-2y)^2 - 10y(6-2y) \right)$$

$$\downarrow$$

$$F_{eq} = \int_0^3 \left(10y^2 - 60y + 90 \right) dy$$

$$F_{eq} = \Big|_0^3 \frac{10}{3}y^3 - 30y^2 + 90y$$

$$F_{eq} = \left(\frac{10}{3}(3)^3 - 30(3)^2 + 90(3) \right) - (0)$$

$$F_{eq} = 90 \text{ lbs}$$

$$X_{eq} = \frac{\int_0^3 \left(\int_0^{6-2y} (30 - 5x - 10y)(x)\, dx \right) dy}{F_{eq}}$$

$$\int_0^{6-2y} 30x - 5x^2 - 10xy$$

$$\downarrow$$

$$\Big|_0^{6-2y} 15x^2 - \frac{5}{3}x^3 - 5yx^2$$

$$\downarrow$$

$$15(6-2y)^2 - \frac{5}{3}(6-2y)^3 - 5y(6-2y)^2$$

$$\downarrow$$

$$-\frac{20}{3}y^3 + 60y^2 - 180y + 180$$

$$X_{eq} = \frac{\int_0^3 -\frac{20}{3}y^3 + 60y^2 - 180y + 180\, dy}{90}$$

$$X_{eq} = \frac{\Big|_0^3 -\frac{5}{3}y^4 + 20y^3 - 90y^2 + 180y}{90}$$

$$X_{eq} = \frac{135}{90}$$

$$X_{eq} = 1.5 \text{ ft.}$$

$$y_{eq} = \frac{\int_0^3 \left(\int_0^{6-2y} (30 - 5x - 10y)(y)\, dx \right) dy}{F_{eq}}$$

$$\int_0^{6-2y} 30y - 5xy - 10y^2 \, dx$$

$$\downarrow$$

$$\Big|_0^{6-2y} 30yx - \frac{5}{2} y x^2 - 10y^2 x$$

$$\downarrow$$

$$30y(6-2y) - \frac{5}{2} y (6-2y)^2 - 10y^2 (6-2y)$$

$$\downarrow$$

$$10y^3 - 60y^2 + 90y$$

$$y_{eq} = \frac{\int_0^3 10y^3 - 60y^2 + 90y}{90}$$

$$y_{eq} = \frac{\Big|_0^3 \frac{5}{2} y^4 - 20y^3 + 45y^2}{90}$$

$$y_{eq} = \frac{67.5}{90}$$

$$y_{eq} = .75 \text{ ft}$$

Solution