Problem **20-R-VIB-DY-16**

This problem, there's a ferris wheel attached at the end of a beam, and the sparrows, we'll have a nice center of mass that rotates, which essentially generates a forest that is not constant in time. It varies sinusoidally with time because the Ferris wheel is rotating the constant angular velocity of 15 radians per second. So essentially, what we're asked to find is, what is the steady states maximum amplitude of vibration? Okay, so we know that this problem deals with a forest sinusoidally Forest vibration of a simple mass and spring system. So even though there's no spring in this system, a beam that when you apply deflection to a beam, you're applying a force, and it travels a set distance, which means that it acts like a spring. Okay? Um, so you can imagine, this system is just being a one D system with a mass a spring. And a sorry, a spring and a force applied to it. Okay, but what this is what you can represent it as so this is a mask, in reality, what this system would look like, is it would deflect. And this here is the distance of deflection, AX. Okay, and you're applying a force to this, based on this Ferris wheel over here, it's rotating. And so the force being applied is varying with time in a sinusoidal matter. Okay, so first of all, what we need to do is we need to find the stiffness k of the system. Okay, so this spring here has a stiffness K. and we are given the mass of this center of mass, and we're given how much the beam deflects from that mass. Okay, so we can find k by simply using f equals to k x, where we know F and we know x, so we can solve for k. So K is going to be equal to f over delta x. So x here is a change in distance, not just a specific distance. And the force is equal to mass times gravity, because it's a gravitational force, divided by delta x, which we're given is 20 millimeters, okay, so this value here is going to be 20 kilograms, times 9.81 meters per second squared divided by delta x, which is 0.02 meters, I change it from millimeters to meters. Okay, and this is equal to 9810 Newtons per meter. And again, depending on what gravity here, you might have some more, some more accurate, more accurate k m, but essentially, this is the spring constant of this system, then we, so we've solved for k. Now what we need to find is the maximum amplitude at steady state. So we know that based on the system, we know how to solve this system, it's um, it's just a we can find the differential equation itself with this system. And there's two parts to the solution, the steady states and the transient part. Okay, so in this case, we're only interested in the steady state or the particular solution, which deals with the forcing frequency, okay. So, we know that the amplitude, the particular solution, part of the amplitude, is going to be equal to f naught not over k divided by one minus omega naught over omega n squared. Okay, times sine of omega naught t. Okay. Now since we're only interested in the amplitude, we're just going to take this term. So x max at steady state is equal to m is equal to the absolute value of f naught over k divided by one minus omega naught over omega n squared. Here, we don't care about the sine term, because we're assuming that one. Okay, and we just take the absolute value in case it's a negative number. Okay, so now here, we can plug, we well, we could try and plugging things in, we have k, but we don't have f naught. And we don't have the natural frequency. Okay, so first, we can find the natural frequency. So the natural frequency always just depends on the mass and spring constant, it doesn't depend on the forcing frequency. Okay, so we know that omega n is equal to the square root of Okay, over M, which is equal to the square root of 9810 Newtons per meter, divided by 20 kilograms, which is equal to 22.147 radians per second. So now we have our omega and omega naught, here we are given this 15 radians per second, that's the forcing frequency, we have k, we just need to find f naught. Now, what is the magnitude of that force that's acting? So imagine you have an E centric mass over here. And then we drive in a different color, an E centric mass over here, we're only interested in the force that is in this direction, okay? Because a force that is in the x direction doesn't cause this beam to displays up and down. Okay? And we're assuming that this beam can only displays up and down, it can't be stretched the left or right. Okay. So when this thing rotates with the set omega, we have a force that we have an acceleration on this eccentric mass, and we have a force on it. Okay. So this force is, essentially, we are only interested in the downwards component of this force. And since it's due to the Omega, we have Mr omega squared. Okay, so that's the centripetal force. So f naught is equal to m r omega squared. Okay. and we are given all of these, so we can find the magnitude of the force, right? So the force is going to be equal to and this is the M, the mass of the interest of the eccentric mass, okay, not the mass of the beam. So this is five kilograms times the radius, which is 0.15 meters times omega squared, which is 15 radians per second squared. So f naught is going to be equal to 168.8 Newton's, okay. And again, this is because as this thing rotates, we have an eccentric mass that is rotating on that generates a force, and we're only interested in the vertical component of the force. Okay, so we only care about the force that is going that way. We don't care about the force that is going out that way. Okay. And so if you think about this last one is rotating. This is a different time spot on here. We're going to only care about this force here, not this force here. Okay, and so that's why this is going to be sinusoidal. Because these two points here are going to vary by 90 degrees. And so when you have full force here, you have zero force over here. Okay? And again, this is just the formula for centripetal force. Okay, so we have all The numbers that we can plug into here. So we can let me go back to black. So we can plug everything in. So the absolute value of f naught, which we said was 168.8 Newton's, and six 8.8 Newton's divided by k, which we calculated to me 9810 Newtons per meter divided by one minus, then we have omega naught, which we're given is 15 radians per second divided by omega n, which we found 22.147 radians per second, and this whole term is squared. And then we again take the absolute value of it, we get that this is equal to m 0.0318. And that's the amplitude. Okay? So x is equal to 0.0318 meters, which is equal to 3.18 centimeters and that is the solution to this problem.