A bar of length 1.5\,m mass of 2\,kg is pinned to the ceiling. A spring, \( k = 50\,N/m \), is attached to the bottom of the bar and a damper, \( c = 10\,Ns/m \), is attached halfway down. Given a small angle displacement, find the damped frequency and the roots.
\[
\frac{c_e^2}{4} \theta + (k_e^2 + mg) \theta + \frac{1}{3} w e^2 \ddot{\theta} = 0 \quad \Rightarrow \quad \text{Eqn}
\]

\[
\dot{\theta} + c' \dot{\theta} + k \theta = 0
\]

\[
c' = \frac{c_e^2}{4} = \left(10 \frac{\text{m}}{\text{s}^2}\right) \left(1.5 \text{m}\right)^2 = 5.625
\]

\[
k' = k_e^2 + mg = \left(50 \frac{\text{N}}{\text{m}}\right) \left(1.5 \text{m}\right)^2 + (2\times 9.81 \text{m/s}^2) = 132.12
\]

\[
w' = \frac{1}{3} w e^2 = \frac{1}{3} (2\times 9.81 \text{m/s}^2) = 1.5
\]

\[
w_n = \sqrt{\frac{k'}{w'}} = \sqrt{\frac{132.12}{1.5}} = 9.385 \text{ rad/s}
\]

\[
w_d = w_n \sqrt{1 - \xi^2}
\]

\[
\xi = \frac{c'}{2 \sqrt{k' w'}} = \frac{5.625}{2 \sqrt{1.5 (132.12)}} = 0.2
\]

\[
w_d = (9.385 \text{ rad/s}) \sqrt{1 - 0.2^2} = 9.196 \text{ rad/s}
\]

\[
w_d = 9.196 \text{ rad/s}
\]

\[
f_{1/2} = -\frac{c'}{w'} \pm i \frac{\sqrt{4w' k' - c^2}}{2w'} = -3.75 \pm i 9.176
\]