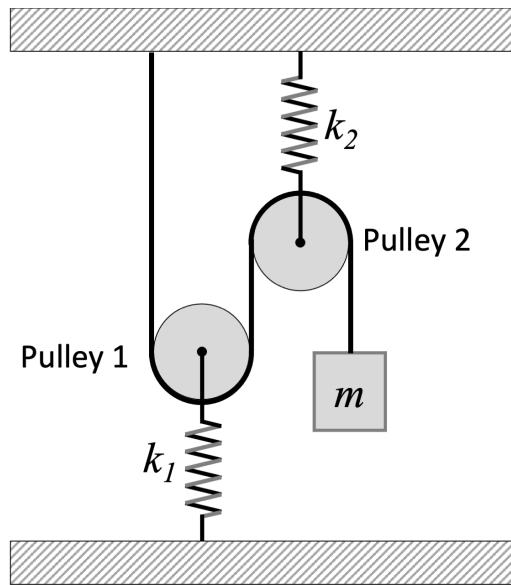


Problem 1 [10 marks] Find the natural angular frequency, ω_n , for the system shown. The pulleys are frictionless and have negligible mass. Values are: $m = 8 \text{ kg}$, $k_1 = 300 \text{ N/m}$, $k_2 = 400 \text{ N/m}$.



Rope has same tension (no stretch, frictionless pulleys).

FBD mass unstretched

$$\begin{aligned} \text{equilib. } T &= k_{\text{eq}} x \\ T &= k_{\text{eq}} \Delta \\ mg &= k_{\text{eq}} \Delta \\ \ddot{x} + \frac{k_{\text{eq}}}{m} x &= 0 \\ \omega_n^2 &= \frac{k_{\text{eq}}}{m} \end{aligned}$$

FBD pulley 1

$$\begin{aligned} \sum F_x: 2T + F_{S1} &= 0 \\ F_{S1} &= -k_1 x_1 \\ \Rightarrow 2T &= k_1 x_1 \end{aligned}$$

$(m=0)$

$$\begin{aligned} \sum F_x: 2T + F_{S1} &= 0 \\ F_{S1} &= -k_1 x_1 \\ \Rightarrow 2T &= k_1 x_1 \\ \text{movement of rope} &= 2x_1 \end{aligned}$$

FBD pulley 2

$$\begin{aligned} \sum F_x: 2T + F_{S2} &= 0 \\ F_{S2} &= -k_2 x_2 \\ \Rightarrow 2T &= k_2 x_2 \end{aligned}$$

$(m=0)$

$$\begin{aligned} \sum F_x: 2T + F_{S2} &= 0 \\ F_{S2} &= -k_2 x_2 \\ \Rightarrow 2T &= k_2 x_2 \\ \text{movement of rope} &= 2x_2 \end{aligned}$$

$$\begin{aligned} X &= 2x_1 + 2x_2 \\ \frac{X}{k_{\text{eq}}} &= 2\left(\frac{2x_1}{k_1}\right) + 2\left(\frac{2x_2}{k_2}\right) \end{aligned}$$

$$k_{\text{eq}} = \frac{1}{4} \frac{k_1 k_2}{k_1 + k_2}$$

$$\begin{aligned} \omega_n &= \sqrt{\frac{k_{\text{eq}}}{m}} \\ &= \sqrt{\frac{1}{4m} \frac{k_1 k_2}{k_1 + k_2}} \end{aligned}$$

$$\boxed{\omega_n \approx 2.31 \text{ rad/s}}$$