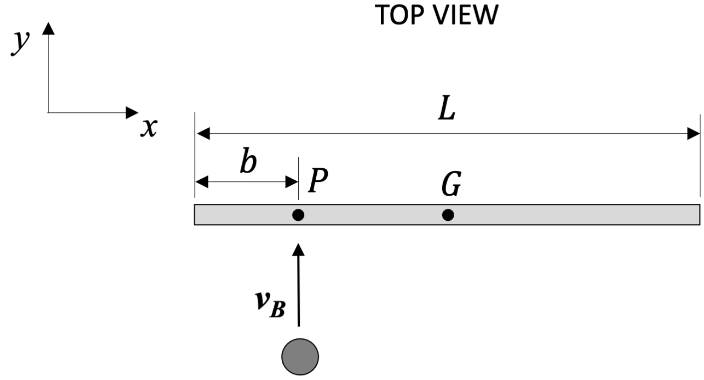
State 1

A rod (m_R = 5 kg, L = 3 m) rests on a frictionless surface. A ball (m_B = 1 kg) with an initial velocity $\overrightarrow{v_B}$ ($\vec{v}_{B1}$) = 12 m/s in the direction shown impacts the rod at P. Find the rod's angular velocity, $\vec{\omega}_R$, and linear velocity of the centre of gravity, $\overrightarrow{v_G}$, immediately after impact if b = 1 m. Assume e = 0.6.

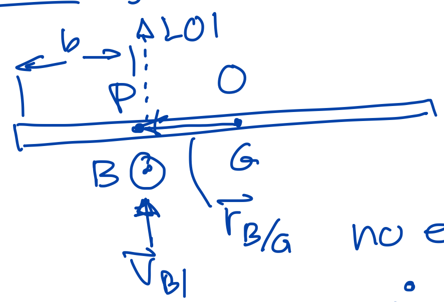
State 2

TOP VIEW



Find $\vec{\omega}_{R2}, \vec{V}_{RG2}$

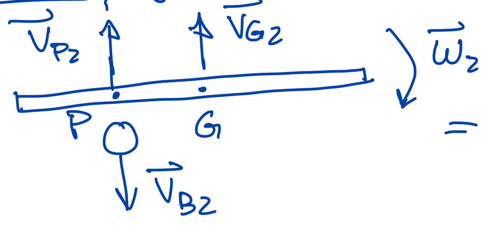
state 1 just before impact



$\vec{V}_{G1} = 0$

$\vec{\omega}_{R1} = 0$

state 2 just after impact



no external impulses

$\therefore$ cons. of momentum for system

$\vec{K}_G$:

$$\sum_{sys} \vec{K}_G \text{ conserved}$$

$$\sum_{sys} \vec{K}_{G1} = \cancel{\vec{K}_{G1R}}^{\,0} + \vec{K}_{G1B}$$

$$= \cancel{I_B \vec{\omega}_{B1}}^{\,0} + \vec{r}_{B/G} \times m_B \vec{V}_{B1}$$

$$= \left(\frac{L}{2} - b\right)(-\hat{i}) \times m_B V_{B1} \hat{j}$$

$$= -m_B V_{B1} \left(\frac{L}{2} - b\right) \hat{k}$$

$$\sum_{sys} \vec{K}_{G2} = \vec{K}_{G2R} + \vec{K}_{G2B}$$

$$\vec{K}_{G2R} = I_{GR}(-\omega_2 \hat{k})$$

$$\vec{K}_{G2B} = +m_B V_{B2} \left(\frac{L}{2} - b\right) \hat{k}$$

$$\Rightarrow -m_B V_{B1} \left(\frac{L}{2} - b\right) = -I_{GR} \omega_2 + m_B V_{B2} \left(\frac{L}{2} - b\right) \quad (1)$$

$\underline{\underline{e}}$: $e = \dfrac{\Delta V_{sep}}{\Delta V_{clos.}} = \dfrac{V_{P2} - (-V_{B2})}{V_{B1} - 0} = \dfrac{V_{P2} + V_{B2}}{V_{B1}} = 0.6$

$\Rightarrow \quad V_{P2} + V_{B2} = 0.6\, V_{B1}$ (2)

2 eqns
3 unknowns
$(\omega_2, V_{B2}, V_{P2})$

$\underline{\underline{\vec{J}}}$: linear mom. conserved for system in y-dir
(impact impulse is internal)

$$\sum_{sys} \vec{J}_{y1} = \sum_{sys} \vec{J}_{y2}$$

always COG in lin. mom.

$\uparrow$: $m_B V_{B1} = -m_B V_{B2} + m_R V_{G2}$ (3)

3 eqn
4 unknowns
$(\omega_2, V_{B2}, V_{P2}, V_{G2})$

<u>kinematics</u>:

$$\vec{V}_{G2} = \vec{V}_{P2} + \vec{\omega}_2 \times \vec{r}_{G/P}$$

$$V_{G2}\hat{j} = V_{P2}\hat{j} + \omega_2(-\hat{k}) \times \left(\tfrac{L}{2} - b\right)\hat{i}$$

$$= V_{P2}\hat{j} - \omega_2\left(\tfrac{L}{2} - b\right)\hat{j}$$

$\uparrow$: $V_{G2} = V_{P2} - \omega_2\left(\frac{L}{2} - b\right)$ (4)

4 eqns
4 unknowns $(\omega_2, V_{B2}, V_{P2}, V_{G2})$

<u>solve</u>:

(3)+(4): $m_B V_{B1} = -m_B V_{B2} + m_R\left(V_{P2} - \omega_2\left(\frac{L}{2} - b\right)\right)$

+(2): $m_B V_{B1} = -m_B V_{B2} + m_R(0.6\, V_{B1} - V_{B2}) + m_R \omega_2\left(b - \frac{L}{2}\right)$ ✓

$(m_B - 0.6\, m_R) V_{B1} = -(m_B + m_R) V_{B2} + m_R \omega_2\left(b - \frac{L}{2}\right)$

$$v_{B2} = \frac{1}{m_B + m_R}\left[(0.6m_R - m_B)v_{B1} + m_R\omega_2\left(b - \frac{L}{2}\right) \right]$$

$$= \frac{1}{6\text{ kg}}\left[(0.6(5\text{kg}) - 1\text{kg})(12\text{ m/s}) + 5\text{kg}\ \omega_2 (0.5\text{ m}) \right]$$

$$= 4 + 0.4167\,\omega_2 \quad \text{m/s}$$

$+①$: $$-m_B v_{B1}\left(\frac{L}{2} - b\right) = -I_{G_R}\omega_2 + m_B v_{B2}\left(\frac{L}{2} - b\right)$$

$$-(1\text{kg})(12\text{ m/s})(0.5\text{ m}) = -\frac{1}{12}(5\text{kg})(3\text{m})^2\,\omega_2 + (1\text{kg})(0.5\text{ m}) * (4 + 0.4167\,\omega_2)$$

$$-0.6 = -3.75\,\omega_2 + 2 + 0.208\,\omega_2$$

$$-2.6 = -3.54\,\omega_2$$

$$\Rightarrow \omega_2 = 1.36 \text{ rad/s}$$

$$\boxed{\vec{\omega}_2 = -1.36 \text{ rad/s}\ \hat{k}}$$

③: $$m_B v_{B1} = -m_B v_{B2} + m_R v_{G2}$$

$$(1\text{ kg})(12\text{ m/s}) = -(1\text{ kg})(4 + 0.4167(-1.36\text{ rad/s})) + (5\text{kg})\,v_{G2}$$

$$12 = -4 + 0.567 + 5v_{G2}$$

$$5v_{G2} = 15.43 \Rightarrow v_{G2} = 3.09 \text{ m/s}$$

$$\boxed{\vec{v}_{G2} = 3.09 \text{ m/s}\ \hat{j}}$$