Problem **20-R-IM-DK-15**

In this problem, we have a ball that is hitting this bar, which is about, which can rotate about O. And we have a coefficient of restitution between the bar and the ball. So first of all, this is clearly an impact problem, and conservation of momentum. So what we're going to do is we are going to first write a conservation of linear momentum between our angular momentum between the two time points. So we're going to identify the two time points. So the first time point is when the ball is right, the is right at bar a, but it's not hitting it yet. Okay? So time point one. So when you have your bar and the ball is right there, right before, then time point two is when you have your bar and the ball hits it. Okay. So that's going to be the naming convention. Okay. So we have conservation of angular momentum. So HO, one, is going to be equal to the sum of a h, o two. So again, this is about a, which is the center over here. And one means this time point, and there's two means this time point, so after impact, okay. So let's write down the term that each of these two sides of the equation have. So h o one is going to be equal to m, it's going to have a few terms, what we can have a few times, but some, most of them are going to cancel in this case. Because in this case, before this ball hits this bar, this bar doesn't rotate and doesn't translate at all. Okay, the only thing that's moving is this ball here, which is translating with the linear velocity case, we're only going to have a linear velocity term to that angular momentum, okay? And that will be the linear momentum times the radius, okay? So we have m, and this is the ball, the velocity of the ball, the initial velocity will be B one times L over two, y L over two, because the radius distance is this distance over here, okay? Not the whole L. And so we have linear momentum VB one times the mass of the ball times this radius, and that gives us angular momentum. Nothing else is moving. So that's the only term at time one, then we have h, o two, which is angular momentum at time point two. So after the ball has collided with the bar, and here, we're going to have multiple terms because it's not just the ball moving, the ball keeps on moving while it moves at a different velocity Now, obviously, so it's going to have a different velocity, but it still has linear velocity. But now this whole system starts to rotate. And that's going to add terms to our momentum. Okay, so first of all, let's add that rotation at the translation of that ball. So we have the mass of the ball and b, v, b, this time to because it's a different velocity, the ball doesn't retain its original velocity after the impact, and then again, times L over two because it's located at the same point. But now we also have the inertia of the rod times omega. And this is I'm going to write the Omega two, but this is the only omega involved This is the only angular velocity involved because it's zero at the beginning. Okay, and so this is on these are two sides of the equation, okay? And if we equate them, we get the following equation MBV b one, L over two is equal to m b, b b two, L over two plus i rod which we're going to substitute substitute in, which is going to be equal 1/12 Ml squared times omega two okay. So now here, we have a few of these terms. So this term here we have because we're given DB one, and we're given the mass and like, okay, now we don't know the second vote the velocity of the ball at 10.2, which is right after the impact. But there's a way to find that. And we're obviously trying to solve for omega two. So that that's our unknown. Okay. Now, how do we get this second velocity, or the velocity after the impact while we're given a kind of coefficient of restitution here, and so we can use that to find the linear velocity of the ball after impact. So the coefficient of restitution e is equal to v a two minus v b two over v b one minus V A one. Okay. So in this case, we know that this term here, v one is zero, because the bar does not move initially, and V eight two is going to be a function, it will be B one we're given, it's just three meters per second. And then, so this leaves us the equation in terms of VA two and the B two. Okay, but VA two, we can relate in terms of omega two. Okay, so we can this leave those to the following equation, the coefficient of restitution is 0.8 is equal to VA to VA two is also equal to omega times the radius. So omega two times L over two, okay. And then minus v v two over v v one minus zero. So, db one we're given, it's just essentially over v v one, center that.

Okay, so here, we have all these terms, so we can essentially solve for VB two in terms of omega two, and we can plug this equation into here. So we have this whole equation in terms of omega two, and we can solve for omega two. Okay, so solving this equation, we get the following per second, which is equal to me to 0.8 times three meters per second, negative plus omega two, L over two. Okay, and this is L here, and we can plug that into the above equation. So we have m v one, v b one, L over two equals two MB VB two, which is equal to negative 0.8 times three meters per second, plus omega two times L over two, plus r times L over two again, plus 112 L squared omega two. Now we can plug in all the values and isolate for omega two, and we get the following. So m b one, so that's the mass of the ball. That is equal to six kilograms, then we have a v v one, which is three meters per second times L, which is two meters over two is equal to the mass of the ball 0.16 kilograms, times 0.8 times three meters per second, plus omega two times two meters over two times two meters over two plus plus 112 m and this is the M of the bar. So And a wish is one kilogram times the length of the bar, which is two meters squared times omega two. Okay. Again, as I mentioned, this is an equation in terms of omega two. And if you isolate and sell for omega two, you get the Omega two is equal to 1.75 radians per second. And this is our final answer.