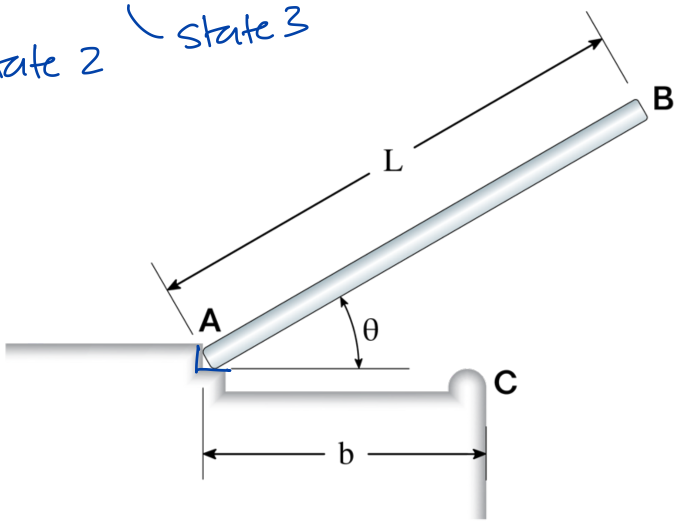
state 1

The rod AB (length L = 2 m, mass 15 kg) falls from rest from an initial angle of θ = 30 degrees. It impacts the corner C (b = 1.3 m). Determine the angular velocity, $\vec{\omega}$, and the velocity of the rod's centre of gravity, $\vec{v_G}$, just after impact.

state 2

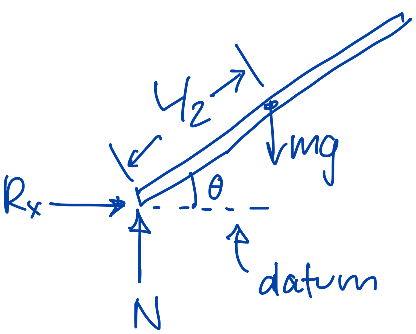
state 3



state 1 from rest

$\vec{V}_{G1} = 0$

$\vec{\omega}_1 = 0$

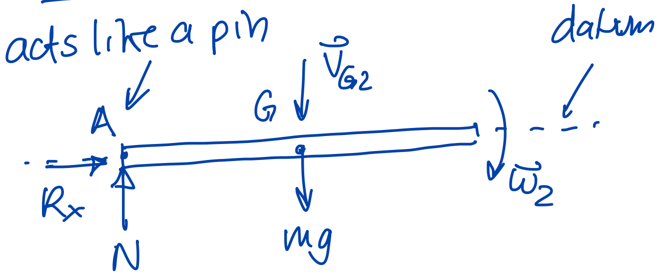


$T_1 = 0$

$V_1 = mg\frac{L}{2}\sin\theta$

conservation of energy 1→2

state 2 just before impact



$V_2 = 0$

$T_2 = \frac{1}{2} I_A \omega_2^2$

$= \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_2^2$

$= \frac{1}{6} mL^2 \omega_2^2$

$$\cancel{T_1}^{\,0} + V_1 = T_2 + \cancel{V_2}^{\,0}$$

$$\cancel{m}g\frac{\cancel{L}}{\cancel{2}}\sin\theta = \frac{1}{\cancel{6}_3}\cancel{m}L^{\cancel{2}}\omega_2^2$$

$$g\sin\theta = \frac{1}{3}L\omega_2^2$$

$$\Rightarrow \omega_2^2 = \frac{3g}{L}\sin\theta \Rightarrow \omega_2 = \sqrt{\frac{3g}{L}\sin\theta} = \sqrt{\frac{3(9.81)\sin 30}{2}}$$

$$= 2.71 \text{ rad/s}$$

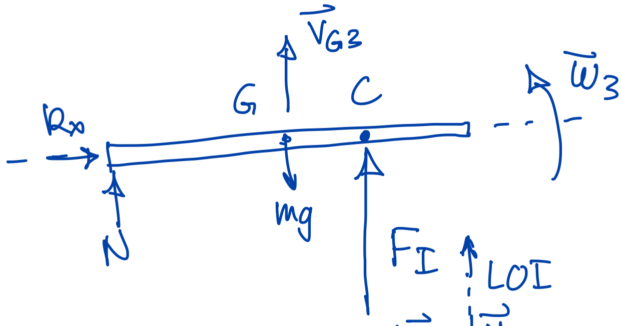
State 3 just after impact

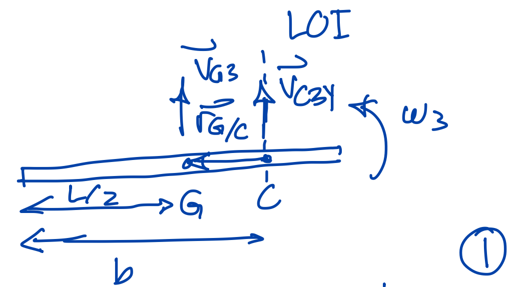
assume $\int W\,dt \ll \int F_I\,dt$

any other force

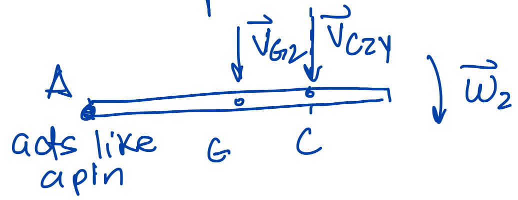
$\therefore$ angular momentum conserved about C ONLY

$2 \rightarrow 3$





$\vec{K}_{C_2}$:



$$e = \frac{\Delta V_{sep}}{\Delta V_{clos}} = \frac{V_{C3y} - 0}{0 - (-V_{C2y})} = 0.4 \Rightarrow \underline{V_{C3y} = 0.4\, V_{C2y}} \quad (1)$$

$$\vec{K}_{C_2} = I_C \vec{\omega}_2 + \vec{r}_{G/C} \times m\vec{v}_{C2}$$

C is not a pin or COG

$$\vec{K}_{C_3} = I_C \vec{\omega}_3 + \vec{r}_{G/C} \times m\vec{v}_{C3}$$

$$\vec{\omega}_2 = -\omega_2 \hat{k}$$

$$\vec{\omega}_3 = \omega_3 \hat{k}$$

$$\vec{r}_{C/A} = b\,\hat{i}$$

$$\vec{V}_{C2} = -b\omega_2 \hat{j}$$

e

$$\vec{V}_{C3} = 0.4\, v_{C2}\, \hat{j}$$

$$= 0.4\, b\,\omega_2\, \hat{j}$$

$$\vec{r}_{G/C} = \left(b - \frac{L}{2}\right)(-\hat{i})$$

$$\vec{K}_{C_2} = \vec{K}_{C_3}$$

$$I_C = \frac{1}{12} mL^2 + m\left(b - \frac{L}{2}\right)^2$$

$$= 6.35 \text{ kg-m}^2$$

$$-I_C\omega_2 + \left(b - \frac{L}{2}\right) m\,(b\omega_2)$$

$$= I_C\omega_3 - \left(b - \frac{L}{2}\right) m\,(0.4\,b\,\omega_2)$$

solve for ω_3

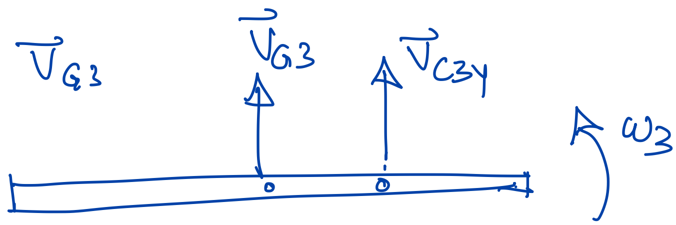
$$I_C\,\omega_3 = I_C\omega_2 + \left(b - \frac{L}{2}\right) m\,b\,\omega_2\,(1 + 0.4)$$

$$\omega_3 = \left(-1 + \frac{\left(b - \frac{L}{2}\right) m b (1.4)}{I_c} \right) \omega_2$$

$$= \left(-1 + \frac{(1.3 - 1\,m)}{6.35\,kg \cdot m^2} (15\,kg)(1.3\,m)(1.4) \right) \underbrace{(2.71\,rad/s)}_{\omega_2}$$

$$\boxed{\vec{\omega}_3 = 0.785\,rad/s\ \hat{k}}$$

Find $\vec{v}_{G3}$



$$\vec{v}_{G3} = \vec{v}_{C3} + \vec{\omega}_3 \times \vec{r}_{G/C}$$

$$= v_{C3y}\hat{j} + \omega_3 \hat{k} \times \left(b - \frac{L}{2}\right)(-\hat{i})$$

$$= 0.4\, b\, \omega_2 \hat{j} - \omega_3 \left(b - \frac{L}{2}\right)\hat{j}$$

$\hat{j}$: $\quad v_{G3} = 0.4(1.3\,m)(2.71\,rad/s) - (0.785\,rad/s)(0.3\,m)$

$$= 1.17\,m/s$$

$$\boxed{\vec{v}_{G3} = 1.17\,m/s\ \hat{j}}$$