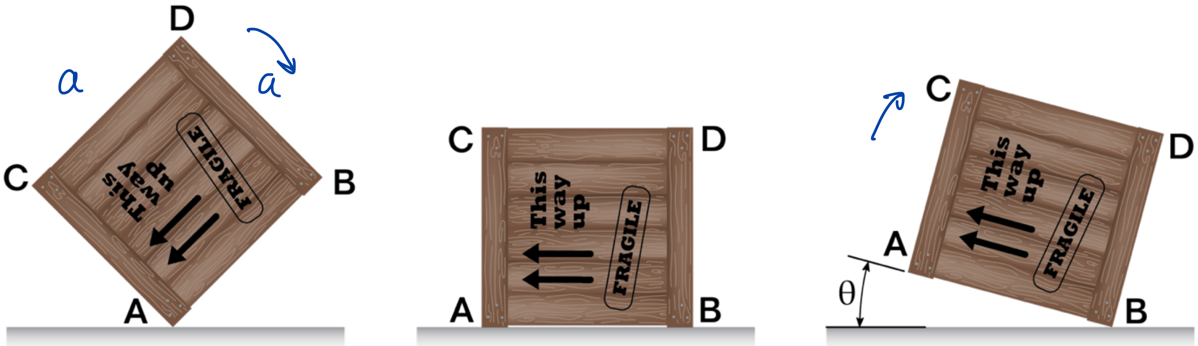
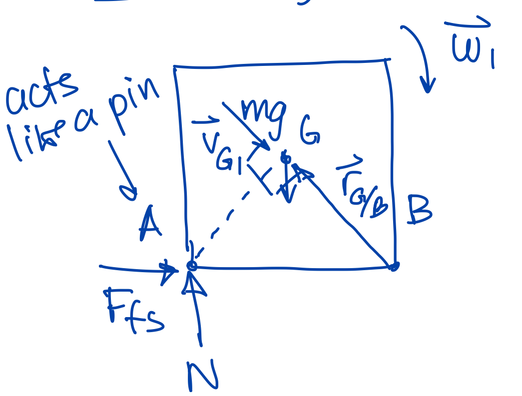
The square crate (dimensions a x a = 0.8 m x 0.8 m, mass m = 20 kg) has an initial angular velocity just before impact of $\vec{\omega}_1$ = 4 rad/s. It impacts the ground at corner B (perfectly plastic impact). Determine the angle, θ, through which the crate will rotate upwards and the percentage of energy lost in the impact. Assume friction prevents slipping throughout.

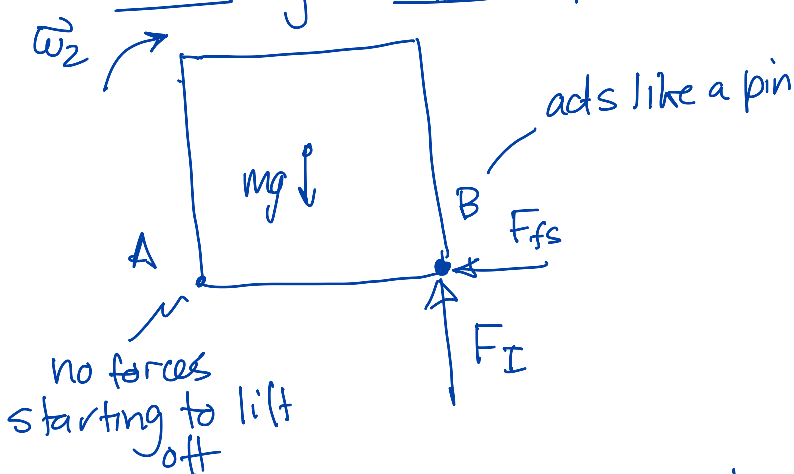


(1) (2) (3)

State 1 just before impact



State 2 just after impact



$\int F_I \, dt >> $ any other impulse in problem (all other impulses negligible)

$\therefore \vec{K}_B$ conserved $1 \to 2$

$$\vec{K}_{B_1} = I_G \vec{\omega}_1 + \underbrace{\vec{r}_{G/B} \times m\vec{v}_{G_1}}_{=0}$$ (vectors in same direction)

$$\vec{K}_{B2} = I_B \vec{\omega}_2$$

cons. angular momentum: $\vec{\omega}_1 = -\omega_1 \hat{k}$, $\vec{\omega}_2 = -\omega_2 \hat{k}$

$$\Rightarrow -I_G \omega_1 = -I_B \omega_2 \Rightarrow \omega_2 = \frac{I_G}{I_B} \omega_1$$

energy loss $1 \to 2$

<u>state 1</u>

$T_1 = \frac{1}{2} I_A \omega_1^2$

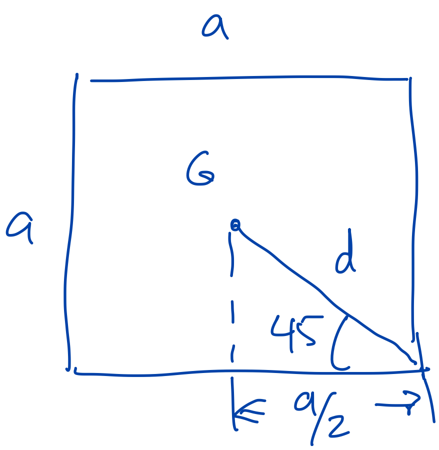
<u>state 2</u>

$T_2 = \frac{1}{2} I_B \omega_2^2$

$I_A = I_B$

$$\Delta T = \frac{1}{2} I_B \omega_2^2 - \frac{1}{2} I_B \omega_1^2 = \frac{1}{2} I_B (\omega_2^2 - \omega_1^2)$$

$$= \frac{1}{2} I_B \left(\left(\frac{I_G}{I_B} \omega_1 \right)^2 - \omega_1^2 \right) = \frac{1}{2} I_B \left(\frac{I_G^2}{I_B^2} - 1 \right) \omega_1^2$$



$d \cos 45 = \frac{a}{2}$

$d = \frac{a}{\sqrt{2}}$

$I_G = \frac{1}{12} m (2a^2) = \frac{1}{6} m a^2$

$I_B = I_G + m d^2$

$= \frac{1}{6} m a^2 + m \frac{a^2}{2}$

$= \frac{1}{6} m a^2 + \frac{3}{6} m a^2 = \frac{4}{6} m a^2$

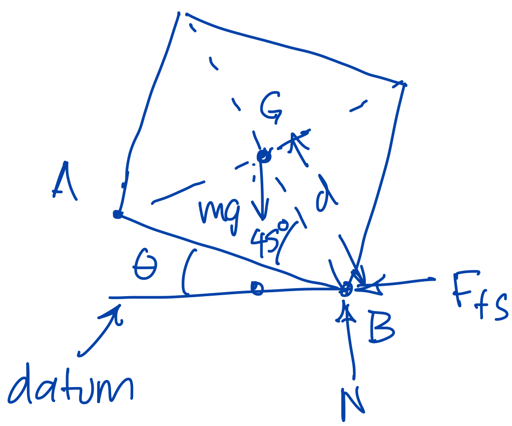
$= \frac{2}{3} m a^2$

$$\frac{I_G^2}{I_B^2} = \frac{\left(\frac{1}{6} \cancel{ma^2}\right)^2}{\left(\frac{2}{3} \cancel{ma^2}\right)^2} = \frac{1}{\cancel{36}_{4}} \cdot \frac{\cancel{9}}{4} = \frac{1}{16}$$

$$\% T_{1 \to 2} : \frac{\Delta T}{T_1} \times 100 = \frac{\cancel{\frac{1}{2} I_B \omega_1^2} \left(\frac{I_G^2}{I_B^2} - 1 \right)}{\cancel{\frac{1}{2} I_B \omega_1^2}} \times 100 = \left(\frac{1}{16} - 1 \right) \times 100$$

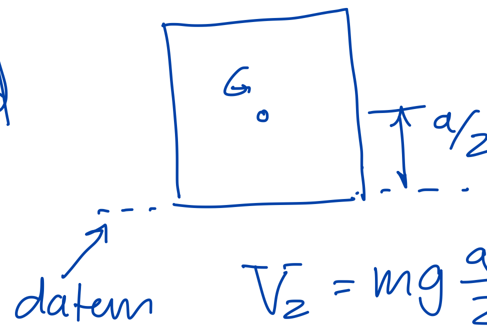
$$\boxed{\% E_{loss\ 1 \to 2} = 93.75\%}$$

state 3 at top of rebound



energy is conserved $2 \rightarrow 3$

recall state 2



$V_2 = mg\frac{a}{2}$

$T_2 = \frac{1}{2} I_B \omega_2^2$

$T_3 = 0$

$V_3 = mg\, d \sin(45+\theta) \qquad d = \frac{a}{\sqrt{2}}$

$\Rightarrow \quad = mg\frac{a}{\sqrt{2}} \sin(45+\theta)$

$$T_2 + V_2 = \cancelto{0}{T_3} + V_3$$

$$\frac{1}{2} I_B \omega_2^2 + mg\frac{a}{2} = mg\frac{a}{\sqrt{2}} \sin(45+\theta)$$

recall: $\omega_2^2 = \frac{I_G^2}{I_B^2}\omega_1^2 = \frac{1}{16}\omega_1^2 \qquad I_B = \frac{2}{3}ma^2$

$$\frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}\cancel{m}a^{\cancel{2}}\right)\left(\frac{1}{\cancelto{8}{16}}\omega_1^2\right) + \cancel{m}g\frac{\cancel{a}}{\cancel{2}} = \cancel{m}g\frac{\cancel{a}}{\cancelto{\sqrt{2}}{\sqrt{2}}} \sin(45+\theta)$$

$$\frac{1}{24} a\omega_1^2 + g = g\sqrt{2}\sin(45+\theta)$$

$$\Rightarrow \sin(45+\theta) = \frac{1}{g\sqrt{2}}\left(\frac{1}{24}a\omega_1^2 + g\right) = \frac{1}{\sqrt{2}(9.81)}\left(\frac{1}{24}(0.8)(4)^2 + 9.81\right)$$

$$= 0.7455$$

$$45+\theta = 48.21° \Rightarrow \boxed{\theta = 3.21°}$$