# Problem **20-R-IM-DK-7**

In this question, we have a fixed gear that is rotating, and it's pushing along a rack, we're given the mass of both the gear and the rack and the radius of gyration of the gear. And we're asked to find what is the moment that we input into the gear if we reach an Omega 20 radians per second in 0.6125 seconds. So this is a momentum impulse and momentum question. So what we need to do first is we need to draw our freebody diagram in order to determine the forces and moments that we're applying to the system. So we're going to start with the rack first. So I'll just draw the rack as a rectangle, but so I'm going to ignore the teeth. But know that there are teeth on the rack. And so on the rack, we have a normal force up. And we have a force that is pushing the rack. So in this case, this whole system is rotating with an Omega that is clockwise on so if we look at the point where these two where the racking and gear meet, on the velocity will be along the negative x direction. So this is going to be V, and I'm going to draw my coordinate system, so x, that way and y is up. So if the velocity, those two points share the same velocity, so the velocity will be backwards, so in the negative x direction, so the force that we need to apply to the rack in order to start from rest, and then move to that direction, is going to be in the negative x direction. And so this is going to be F, which is the force that we that the gear and the rack transmit between each other. And then we have been, of course, the gravity, obviously, of the rack. And since there's no friction, we do not have a friction force. Okay, so this is for the rack. And then we have the gear. So the gear again, I'll just draw it as a circle, but know that it does have teeth. So the gear again, it's gonna have its force due to gravity, fgx at the center, it also has that same exact force that we are applying because this force is equal and opposite between the rack and the gear. So this is again, the same force F, then we also have a moment, which we're applying, and the moment that we apply is going to be in the direction of omega. so omega will be in this direction. So this is also the for the direction of the moment, because we're accelerating everything in that direction. So that's the direction we applied the moment in. And sorry, this worse, should be in the opposite direction, because it's equal and opposite to the force from the rack over here, okay. And then we have our reaction force over here. gx and gy, y. And those are the reaction forces because this gear here is pinned at the center. Okay. Now, we're not interested in all of these forces, we're just interested in a few. And that's why this is why we need a freebody diagram to identify which forces we need to take into account in the with momentum. Okay, so first of all, we have to figure out what this velocity that I drew in green here is, so remember the velocity v. Sorry, let me write it in black. V is equal to omega cross our a, so in this case, we have omega and omega and so this, we're looking at the end, okay, so in the beginning, the velocity zeroes omega zero, so everything is fixed at the end, so at the end points after 0.6125 seconds. We have omega 20 radians per second, and we're trying to find what is this velocity down here. So using the vector equation, we have our radius or R, which is starts from here and points down that way. This is r and omega is going to be in the k hat direction because it's rotating in the xy plane, and it's rotating clockwise, so it's going to be in the negative k direction, because we assumed a positive rotation to be counterclockwise. Okay, so if we plug everything in, we get the following omega being 20 radians per second in the negative k hat direction, cross product to our, which is going to be negative 0.15 meters in the j hat direction, and this will give us a velocity of negative three meters per second in the i hat direction. Okay. So this is going to be our velocity V, which I forgot to add the vector sign on top. And just to make it clear, this here is our Okay, so now that we have our velocity, we can actually do momentum balance for both the rack and the gear. So let's do for the rack first. So let's write momentum balance. So first, we're going to do the rack. For the rack, we're going to look at the two time points. So this is the general formula applied to this rack, we have the mass, this rack is not rotating, it's just translating. So we don't have to take into account any omega, we only have to take into account linear velocity, so we have m of the rack times the velocity of the rack at time point one plus the sum of the integral from t one to T two, F in dt equals to V RAC of two. So again, this term here is at the beginning, and this is at the end. And this is actually to get from the beginning to the end. But this is going to be equal to zero because the initial velocity of the rock is zero. And then we have a mass times the velocity of rap two wishes, what we just found over here, the final velocity of the rack. And then there's this term here, which is the integral from tier one to tier two, or courses in dt. Now, in this case, we're going to assume that all the forces are constant. And so we can actually take out this term from the integral because they do not depend on time, and then we just multiply it by the time interval, because the integral from t one to T two of dt is just at that time interval. And so that makes it simple because the forces are going to be constant throughout the whole time. And again, this here, there's no omega terms, no rotational velocity terms, and there's no moment terms, because there's no woman supplied to the rack, it's just linearly translating. Okay? So we can actually plug things in here. So this first term goes to zero, because the mass of the rack, so this is zero meters per second, the velocity of the rack at time one is going to be zero, and the mass of the rack is 20 kilograms. Plus, there's only one forest that we're interested in here, on. So if we go back to our freebody diagram, the only one force that we're interested in is F. Because all of the other forces. Yeah, they equal each other, they cancel. So the only force that we're interested in is the force F. And this force f is going to be equal to, we can pull it out of the integral because we know it's constant, right? And it doesn't depend on time. So this is just going to be equal to AP, because we call this force up here. And times the time the integral from t one to T two of dt, which is essentially the time interval, so times t, which is going to be equal to the mass of the rock again, 20 kilograms, times the velocity of rock to which we just found up here was three meters per second. Okay, now we can actually plug in t because we're given T is 0.6125. And we can solve for F. So given that t is 0.6125 seconds, we solve for F and we get that f is equal to 60 Newton's, okay, so that force there is going to be equal to 60 Newton's. Now let's go and do the same thing for the gear. Okay, so for the gear here, you can see that we're going to have this force F, and this moment M. So we're gonna the same equation holds, it's just, we don't have linear velocities anymore, we have wrote my angular velocities. And we have a moment here, okay. And this moment, is going to be created by both the moment that we're applying, but also this force here. Okay, so we also have to keep into account that force. And that's two unknowns, but we've already solved for this for us. And so it actually gives us one unknown the moment. Okay, so let's do the same thing. So we have, instead of m v, we have ag omega, so we have i g omega one plus the sum of the integral from time one to time to have the moment in about GNdt, which is equal to Ig, omega two. Okay. So again, just like before, this term here is zero, because everything starts from rest is not rotating. So we have Igy, which we can get from the radius of gyration. So we have a radius of gyration of 125 millimeters. So we can actually solve for, for IG with M kg squared. And then multiplied by omega, which is zero radians per second. And then here, we're going to have M, which is 30 kilograms. And then kg is 0.125 meters squared. Okay. So this, let me just write it out neatly. This here is my G. Because this is m, times the radius of gyration squared, and remember, this radius of gyration are given in millimeters, we have to make it into meters to get standard units. And then this gives us zero, so that term all cancels. Plus, again, that moment that we're applying here in the clockwise direction, is going to be positive, but this moment, is going to be constant throughout time. So we can pull it out of that integral and say m times the time interval, okay, then we also have another moment that is created by this force down here. And this course down here is moving everything in the counterclockwise direction. So that's going to be opposite, okay, so it's going to have an opposite sign of this, it's going to be in the opposite direction. And this moment here is going to be the force times this radius here, which is the radius of that gear that circle. So that's going to be equal to minus f times R, again, times the integral because we're assuming that f is constant through time radius is constant throughout time, we can pull that out of the integral, and we can times it by the time and this is going to be equal to i g omega two, which is again, the same Igy 30 kilograms times 0.125 meters squared times omega two, which we're given is 20 radians per second. Okay, so here, we can plug in F, we can plug in our because we're given and we can also plug in t because we're given all of this. And so you get the following. M is going to be equal to 30 kilograms, times 0.125 meters squared times 20 radians per second, plus f which is 60 Newton's times R, which is 0.15 meters times t. Which is 0.61 to five seconds divided by t which is 0.61 to five seconds and we get the M is equal to 30 Newton meters. And that is our final answer.