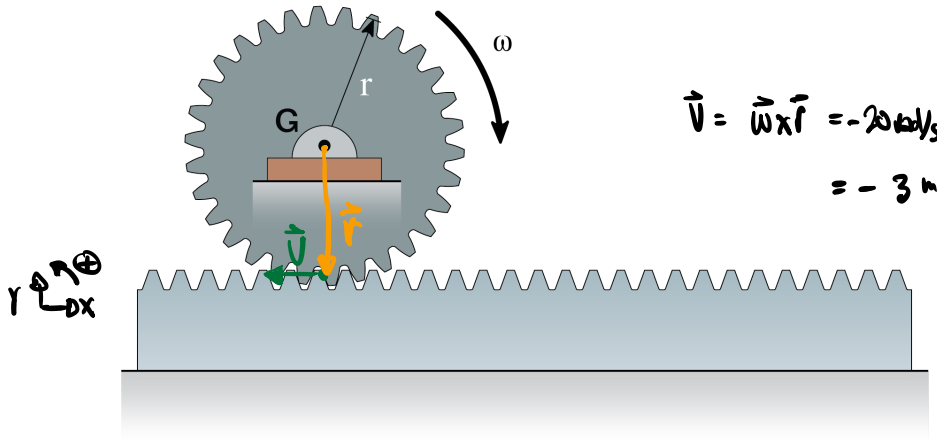
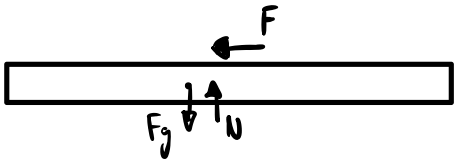
A city engineer is working on a cargo transport system utilizing gears and racks. She is considering a $30kg$ gear with a radius of gyration of $k_G = 125mm$ and a radius of $r = 0.15m$. The gear is in contact with a $20kg$ rack. If it takes the gear $0.6125\ sec$ to reach an angular velocity of $\omega = 20\ rad/s$, starting from rest, determine the moment that the gear is subjected to. Assume there is no friction between the rack and the ground.



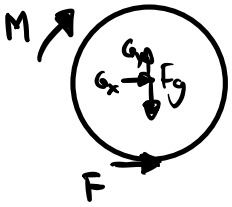
$\vec{v} = \vec{\omega} \times \vec{r} = -20 rad/s\,\hat{k} \times -0.15\hat{j}$

$= -3\ m/s\,\hat{i}$

Rack:



Gear:



Momentum Balance:

→ Rack:

$$m(v_{rack,1}) + \sum \int_{t_1}^{t_2} F\,dt = m(v_{rack,2})$$

$$(20kg)(0\ m/s) + F\,t = (20kg)(3\ m/s)$$

$t = 0.6125s$

$$\therefore F = 60N$$

→ Gear: $I_G \omega_1 + \int_{t_1}^{t_2} M_G \, dt = I_G \omega_2$

$$\underbrace{(30\text{kg})(0.125\text{m})^2}_{I_G}(0\text{rad/s}) + Mt - Frt = (30\text{kg})(0.125\text{m})^2(20\text{rad/s})$$

$$M = \frac{(30\text{kg})(0.125\text{m})^2(20\text{rad/s}) + (60\text{N})(0.15\text{m})(0.6125\text{s})}{(0.6125\text{s})}$$

$$\boxed{M = 30\ \text{N}\cdot\text{m}}$$