Problem **11**

In this problem, we have a race car that is traveling along a straight road with a constant speed of 80 kilometers per hour. The engine provides a torque of 50 Newton meters at each of the back two wheels that have a radius of 0.5 meters, we are asked to find what is the output power of the motor. And then with this output power, what is the efficiency of the motor if the input power is 17.78 kilowatts. So let's first focus on the first part of the question, which is finding the output power of the motor. So we can see that we're given a torque at each wheel. And we're given a velocity of the car. So we know that the power p is equal to the torque times the angular velocity omega. So each of these wheels, we're going to have an input torque, and these wheels are spinning with a certain omega. So that's going to determine our input power, or our output power. So this is the power that we get out of the system to propel the car forward. Right? This is, essentially, since we're assuming there's no friction at the wheels, no losses, all of this power will be used to propel the car forward. So we have the input torque, which is 50 Newton meters, but we do not have the angular velocity. But we can find the angular velocity, given the wheel dimensions and the velocity of the car. So we're assuming there's no friction between the road and the wheels. So our instantaneous center of zero velocity will be located at the bottom of the wheel. Here where the axle connects the wheel, we know that this is going to have the velocity of the car, right, which is 80 kilometers per hour. So given these two points and velocity being 80 kilometers an hour there and zero over there, we can determine the angular velocity of the wheel. And from this angular velocity, we can determine the output power of the system. So let's draw our wheel. And we draw the instantaneous center of zero velocity there, this is where the axle connects to the car. So we know that this point is going to have a velocity v equal to 80 kilometers per hour. And this here, this distance here is simply the radius of the wheel, which we are given, so r is equal to 0.5 meters. And then we have r omega, which is what we're trying to determine the angular velocity. So we know that the velocity vector v is equal to the angular velocity vector crossed to the radius vector. Right. Now, if you look at this diagram, here, we can see that all these vectors are perpendicular, right 90 degrees apart. So the velocity points to the right, the radius points upwards, and angular velocity points either into or out of the page, depending on the direction, right, but they're all perpendicular. So we can get rid of this vector equation, get rid of this cross product and actually just turn it into a scalar equation. So we can determine that omega is equal to v divided by r. And again, I do not have the vector symbols anymore, because I know that these vectors are all perpendicular. So this cross product turns into a multiplication of the magnitudes. So now that we've solved for omega over here, we can plug it into this equation to determine the output power. So P out is equal to torque out and omega of the wheels, right? So this is going to be equal to T times V over r. And this if we plug in the values is going to be equal to 50 Newton meters, times 80 kilometers per hour, divided by 0.5 meters, right? That's the radius. Now, we also have to convert this 80 kilometers per hour into meters per second, because we want to get watts out and then turn it into kilowatts. So we're going to multiply by one over 3.6 hours, meters per kilometers per second. And if we cancel out the unit's we see that this kilometer cancels out with that kilometer is hour cancels out with hour here, and we're left with meters per second. The last thing to do is we need to multiply this by two. And why do we multiply this by two because the question here says, of each of the back two wheels, so we actually have two wheels, so we have to multiply everything by two to get the total power output. So if we plug these values into our calculator, we get that P out is equal to 4.44 kilowatts. And this is the final answer for the first part. This question the output power of the system. Next up, let's look at the efficiency. So, efficiency we know is defined as the output power. So the useful power divided by the input power, the power that we put into the system. So, the output power mode is, most of the times in this we have a perfect ideal system going to be less than the input power because some energy is expended to, to convert some other forms of energy and is lost, right. So, P out is usually smaller than p and if it's equal, we get a perfect efficiency. So, we get an efficiency of 100%, or of one, which means that all the energy that we put into the system is converted into the type of power energy that we want. So, in this case, propelling the car forward, and in real life, obviously, this is never the case. And we can never have an efficiency larger than one because that would mean that we somehow created power, which is again, not possible. So, in this case, we just calculated the output power. So this is the useful power and propelling the car forward we just calculated to be 4.4 kilowatts, and we're given an input power of 17.78 kilowatts. So we can calculate the efficiency of this motor by just dividing the two values, so 4.44 kilowatts divided by 17.78 kilowatts. This gives us an efficiency of 0.25. And this is going to be equal to the efficiency and this is our final answer for the last part of this problem.