Problem **20-R-WE-DK-25**

In this question, there's a pulley with a chain, you're pulling on the chain with a force of 50 Newtons. And you're asked to find what is the final angular velocity after a rotation of 90 degrees, assuming everything starts from rest. So again, this is a work energy problem. So we're going to define two states, the initial state and the final state. The initial state is when the chain is in the initial position with zero velocity. And then there's a final state when the chain is moving, and the pulley is rotating. So that's going to be called state two. And we're going to calculate all of the energies. So kinetic and potential, and also add the work between these two states that you put into the system, we're going to equate them and then solve for whatever unknown we need. Okay, so we're going to start with state number one. And with state number one, this is shown in the diagram to the left, we have lb and LA, so two different length chains on the right and left side, and the force FA is applied to the left side at eight. And so everything, the whole system will rotate, like this. And in the second state, this chain here, I'll draw this in red, this chain here will become longer. So it will extend a bit longer. So this is the change in length of this chain, and this chain here will get shorter. So this is going to be the other chain, okay. And so this is going to be L A, two, and this is going to be L b two, whereas on whatever is drawn in the picture is going to be la one and lb one, so at time one. So everything at time one will be equal to the initial state with zero velocity. At time two, we have omega, we have the chains moving, and these lengths will be different will be short, longer on the left side and shorter on the right side. Actually, this will extend all the way up to here. Okay, so, and I'll draw arrows over here as well. So let's analyze the system. Um, so we're going to start with state number one. So that's the everything drawn in black. And we have the potential energy of the two chains. And then while that's essentially it, there is no kinetic energy, because nothing is moving, everything is starting for rest. Okay, so we have T one equals to zero, and then we have v one is going to be equal to the potential energy of the two chains. So we have, we're gonna start with the chain on the left, and the chain on the left is going to be is going to have the following potential energy. Well, first of all, we're going to set our datum and our datum will be this starting from the pivot, so everything downwards, we will define as negative, but we'll see that doesn't really matter. Okay, so v1 is going to be negative the mass per unit length of the spring, which we'll call kilogram per meter, but this is essentially what we're given in the question. There's 3.4 kilograms per meter, the mass per unit length of chain, times the length of the chain on the left, which is L a one, which again, we're given m times ci, the gravitational constant, and then we're going to multiply it by l, A over two, and this thing An Le one. Okay, so just to break it down. This here is the mass and we have G, and then we have H, mgh. So this is the chain on the left, then we can do the chain on the right, which is again negative because it's downwards. kilograms per meter of the spread of this chain, times lb one times g times lb, one over two. And this one divided by two, remember because the center of gravity is halfway down the chain, so this eight Here is gonna start from our data and go halfway down because it only goes to the center of gravity. Okay, so that's why we have divided by two. So then we can plug all of these values that were given inside, so we have negative 3.4 kilogram per meter, times le one, which is three meters, times 9.81 meters per second squared, times three meters divided by two. And I'm going to go on new line minus 3.4 kilograms per meter, times two meters, times 9.81 meters per second squared, times two meters divided by two. And so we got a final value of b1 being equal to negative 216.8. joules. Okay, so now we have our potential energy, we know kinetic energy is zero. So we can move on to state number two. Okay, so state number two is the state that we've defined in red. Okay, so this is after the 90 degree rotation. And where the, the right chain has gotten shorter, and the left chain has gotten longer, so and then here, we also have velocity because the system is rotating and moving. So let's start with the potential energy. And let's start with the changing length. So I define here, if this system rotates that way, 90 degrees, then we're going to have this chain getting longer. Okay? So this chain will get longer by a certain amount, which is equal to the angle that throating tating pi over two times the radius, okay. And that's essentially going to be, well, that's going to be pi over 10. If you calculate it, so the delta L. And I'll define it in red here, delta L, which is how much the left chain gets longer by, and the right chain gets shorter by okay. So this distance here will be delta l also delta L. Okay. So this delta l here will be equal to theta times the radius, which is equal to theta is 90 degrees. So, pi over two times the radius, which is 0.2 meters. And so this is equal to pi over 10 meters. Okay. So, this, this here will use to calculate the new centers of gravity, which is on the left side a bit lower than the original one. And here, it'll be a bit higher than the original one because the chain moved up. And so we can calculate the new potential energies. Okay. So again, it's the same exact formula as over here. But these lengths here will not be la one and l b one, it'll be Oh a tu and lb two, and we define these as l A to being equal to L a one plus delta L and L b two is equal to L b one minus delta L. Okay? So fairly simple. So we have v2, so the potential energy at two being equal to negative kilogram per meter. So again, this is the linear density of the chain, and then we have times L a two times g times L, a two over two minus a kilogram per meter of L, B to G times l be two over two. Okay. And if we plug everything in, we get the following negative 3.4 kilogram per meter times three meters plus pi over 10 meters, times 9.81 meters per second squared times three meters plus by over 10 meters over two. And this is the first part of the equation that we have continues on minus 3.4 kilograms per meter, times two minus pi over 10 meters, and this is meters also as 9.81 meters per second squared times two meters minus pi over 10 meters over two. Okay, and if you solve this, we get that B two is equal to negative 230.6 joules. Okay. So again, this is the potential energy at state two. Now we can calculate the kinetic energy, so t, so T two is going to be equal to depend on omega, okay. And since there's no slipping omega can also give us the linear velocity of the two chains that are hanging. Okay, so there's going to be three terms to this to the kinetic energy, the first term is going to be due to the from the rotation, the energy stored in the rotation of the pulley. So that's going to be one half i omega squared, and this is I have the pulley, okay. Then there's going to be a term from the translation of chain A, which is one half and a VA squared. And then from chain B, one half, and b, b, b squared. Okay. So first, we need to calculate II, which is the inertia of that pulley, which is just equal to one half r squared, which is equal to one half times 20 kilograms times 0.2 meters squared, which is equal to 0.4 kilograms, meters squared. Okay, so now we have I, we don't have omega, but again, we're solving for omega. Now we need to figure out VA and VB. And since they're attached to the same pulley, they're going to have the same velocity. So VA is going to be equal to VB. And this is also going to be equal to omega cross product to R. But since omega and are perpendicular, we can just simplify to omega times r. and omega is going to be our unknown. R, we know. So if we plug this into VA and VB, this whole equation becomes in terms of omega, and everything else is now okay. And since we're trying to solve for omega, that's, that's what we're looking for. So we can solve for t to being equal to one half times 0.4 kilograms, meters squared times omega squared plus one half times m a. and M A is going to be equal to the linear mass density of kilograms per meter. So 3.4 kilograms per meter times l, A. And this is again at the second instance. So we have to add that pi over 10. Okay. So this is three meters plus pi over 10 meters. And that's our, well that's our mass because we have a linear density times length, which gives just gives us the kilograms and then we have VA, which is omega times R, which is 0.2 meters. Okay, and then we have our last term, which is the, I'll continue on to the next line plus one half times 3.4 kilograms per meter, this time, we have the right side. So it's two meters minus pi over 10 meters, times omega times 0.2 meters. So this is our total kinetic energy. And Oh, I forgot to mention this here, squared, and this year is squared. Okay. And with this, we can solve and simplify this equation in terms of omega squared. So T two is equal to 0.5 for omega squared. Okay. And so this really simplifies our equation. And this is just grouping all of these omega squared terms together, and plugging these numbers into a calculator and adding them. Okay, so now we have our, we have potential and kinetic energy at all of the states. So let me highlight them. So we have potential energy at state two, and we have our kinetic energy at state two, then we have our potential energy at state one, and our kinetic energy at state one. Okay. So when we add them all together, they should equal and we also should add the work that we've added into the system. So T one plus v one, plus the work from one to two, should equal to T two plus v two. Okay. And this is we can just plug everything in, the work that we have already calculated is just from a force or non conservative force, which is just equal to the force times the distance that force travels. So the force is equal to 50 Newtons. And the distance that it travels is just that pi over 10. Write that delta l pi over 10 meters. Because that force essentially travels this little distance over here. So it starts from here and ends up here. So travels that delta L. Okay, so pi over 10 meters. And if we put everything together, we get the following. Zero minus 216.8 joules plus 50 Newton's times pi over 10 meters, is equal to 0.54 omega squared minus 230 point six joules. And if we solve for omega squared, we get that omega is equal to 7.4 radians per second. And this is the final answer.