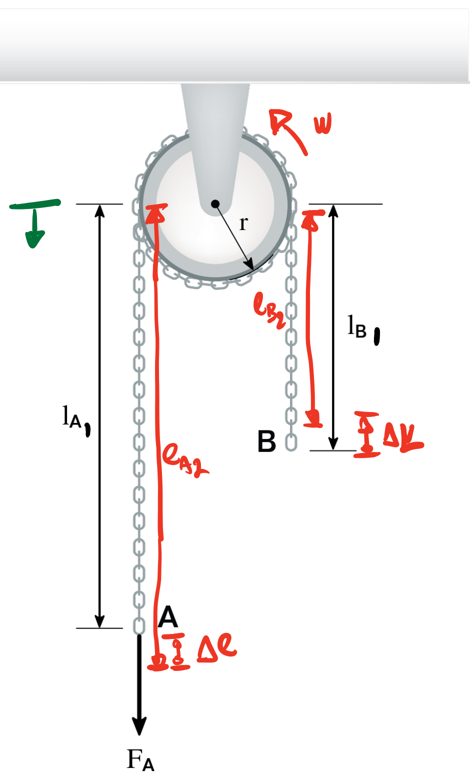
For a summer, you've taken a job at your uncle's auto shop. You pull on the left side of a chain wrapped around a pulley with a force of $F = 50\ N$. The pulley has a mass $m = 20\ Kg$ and a radius $r = 0.2\ m$. If the chain has a mass of 3.4 Kg/m, determine the angular velocity of the pulley after it has rotated $\theta = 90\ degrees$. There is $l_A = 3\ m$ of chain hanging off the left side and $l_B = 2\ m$ hanging off the right side of the pulley. Assume the chain does not slip and that the system was released from rest just before you pulled on it. Assume the pulley can be modelled as a disk.



UBC Engineering

State #1

$$T_1 = 0\ J$$

$$V_1 = -\underbrace{\left(\frac{kg}{m}\right) l_{A1}}_{m}\, \underbrace{g}_{g} \underbrace{\left(\frac{l_{A1}}{2}\right)}_{h} - \left(\frac{kg}{m}\right) l_{B1}\, g \left(\frac{l_{B1}}{2}\right)$$

$$V_1 = -\left(3.4\frac{kg}{m}\right)(3m)(9.81\ m/s^2)\left(\frac{3m}{2}\right) - (3.4\ kg/m)(2m)(9.81\ m/s^2)\left(\frac{2m}{2}\right)$$

$$V_1 = -216.8\ J$$

State#2

$$\Delta l = \theta r = \left(\frac{\pi}{2}\right)(0.2m) = \frac{\pi}{10}\ m$$

$$l_{A2} = l_{A1} + \Delta l \quad ; \quad l_{B2} = l_{B1} - \Delta l$$

$$V_2 = -\left(\frac{kg}{m}\right) l_{A2}\, g \left(\frac{l_{A2}}{2}\right) - \left(\frac{kg}{m}\right) l_{B2}\, g \left(\frac{l_{B2}}{2}\right)$$

$$= -\left(3.4\,\frac{kg}{m}\right)\left(3m + \frac{\pi}{10}m\right)\left(9.81\,m/s^2\right)\left(\frac{3m + \frac{\pi}{10}m}{2}\right)\ldots$$

$$-\left(3.4\,\frac{kg}{m}\right)\left(2m - \frac{\pi}{10}m\right)\left(9.81\,m/s^2\right)\left(\frac{2m - \frac{\pi}{10}m}{2}\right)$$

$$V_2 = -230.6\,J$$

$$T_2 = \frac{1}{2}I\omega^2 + \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$I = \frac{1}{2}mr^2 = \frac{1}{2}(20\,kg)(0.2m)^2 = 0.4\,kg\,m^2$$

$$v_A = v_B = \vec{\omega} \times \vec{r} = \omega r$$

$$T_2 = \frac{1}{2}(0.4\,kg\,m^2)\,\omega^2 + \frac{1}{2}\left(3.4\,\frac{kg}{m}\right)\left(3m + \frac{\pi}{10}m\right)\left[\omega(0.2m)\right]^2\ldots$$

$$+\frac{1}{2}\left(3.4\,\frac{kg}{m}\right)\left(2m - \frac{\pi}{10}m\right)\left[\omega(0.2m)\right]^2$$

$$T_2 = 0.54\,\omega^2$$

$$T_1 + V_1 + U_{1\to2} = T_2 + V_2$$

$$U_{1\to2} = Fd = (50N)\left(\frac{\pi}{10}m\right)$$

$$0 - 216.8\,J + (50N)\left(\frac{\pi}{10}m\right) = 0.54\,\omega^2 - 230.6\,J$$

$$\hookrightarrow \boxed{\omega = 7.4\,rad/s}$$