Problem **20-R-WE-DK-5**

In this problem, we're asked to find the total kinetic energy of the mechanism that has three slender Rods, and we're given the initial omega of AB. And the system is constrained in the following way. So since we're given omega AB, and we need to find the kinetic energy, we need to find omega for each of the sections, because that's how you find kinetic energy, need the velocities or angular velocities. And since we're given omega AB, we start here we find the velocity of B, and then we can find velocity of C and omega BC, then we can find omega C, D, with ec. Okay. So again, whenever you have to find the kinetic energy, you need to look at velocities. And we start from the point where we know the velocity, which is rod AB. So first of all, I'm going to write in my coordinate system, so x is positive to the right, y is positive up, and rotation is positive counterclockwise. And then I will just start with rod AB. So we can use our velocity equation, which is VB, relating velocities in two points along a rigid body is equal to V A plus omega AB cross product to our A, B with respect to A. So in this diagram here, VB will be a vector that is, again perpendicular to rod AB. So this is going to be VB. And then we're going to have our rate or radius or distance r, v with respect to A, which is which links point A and point B. So remember, starts at a and that B, so this is our of B with respect to A. Okay, and we can plug in values, because again, we have omega A B, we have our V with respect to A and VA is zero, to find VB, and VA is zero, because A is pinned, so no motion. So we can solve for VB by just doing the cross product of omega AB, which is five, but it's going to be negative five radians per second in the k hat direction. Remember, negative is important because we defined rotation to be positive counterclockwise, and omit ABS clockwise cross product to our V with respect to A, so this is going to have two components based on that theta angle. So on the x component is going to be negative 0.4 times coasts of 30 degrees in the i hat direction, and then the j hat component will be positive plus 0.4. Sine of 30 degrees in the j hat direction. Remember, negative because we're going to the left and then positive because we are going up. So with this, we can solve this cross product and we can solve for VB and VB is going to be equal to two cosine of 30 degrees in the j hat direction, and two sine of 30 degrees in the head direction. This is all of the units are meters per second here, because it's a linear velocity. Okay, so now we have VB. Next, we're gonna move on to a rod BC. So we could use this same equation here. But we have two unknowns, we wouldn't know the omega and we wouldn't know v c. So what we can do is we can relate everything to the instantaneous center of zero velocity, which we can easily find in this case. And once we relate everything to the instantaneous center of zero velocity, we use this equation about the ice said B. This velocity here would be zero, because the velocity at the instantaneous center velocity zero velocity is always zero. And we can directly solve for omega AB, or omega bc given BD that we already know. So we have to find the instantaneous center of zero velocity for this bar over here. And in that case, instantaneous center of zero velocity is remember when you link to the when you have the velocities at the two extremes you for any points, you find you draw a line perpendicular to these two velocities where these two lines cross. That's the instantaneous center of zero velocity. So we know VB is in that direction and we are going to know that VC has to be in the end direction, okay, this is VC. And again, they're all vectors. So given that, we are going to find the instantaneous center of zero velocity by drawing a line perpendicular to the velocities, we draw one over here, it's a bit offset, because I don't want to go over that radius. But we can see that these two lines meet here at the ice Zed v. Okay, so that's the instantaneous center of zero velocity. And we have all those distances, or we can solve for the dimensions or the distance of that point to rod BC. So if we draw the triangle, which kind of looks like that, so the top length is, so this is B, and this is C, the top length is 0.5 meters, then we have this data angle being 30 degrees, so we can solve for the right side, which is going to be root three over six. And with that, we have the x component and the y component of the ice that V, so we can go from here, y component x component related back to B. Okay, so we can write down that same equation for BB BC, about the ICS at V. So we have V, V D, is going to be equal to the velocity of the ICS, that V, which we know is zero, so we can already cross it out, plus omega BC, cross product to our of B with respect to the icees, which we just solve for that, that those lengths. Okay, so VB is known. And this is known, so we can solve for omega VC. And we get the following equation. So VB, I'm just going to copy two sine of 30 degrees in the i hat direction, plus two cosine of 30 degrees in the j hat direction, I just switched the two components. So I j, j, is going to be equal to omega BC, which is unknown. So we're going to leave it at omega BC, but we know it's going to be in the k hat direction. Because it can only be in the k hat direction cross product, our V with respect to ICS, AED V, which is essentially this length here. So again, I'll draw it offset even more, but everything should go along. Rod, AV, I just don't want to put everything on top of each other. So you can see it. So our b with respect to ICS and a B. And so that's essentially just this hypotony is but we want to have it in components. So we have this is the y component and the positive y and this is the x component and the negative x, so that's going to be negative. So we have negative 0.5 meters in the i hat direction, plus root three over six meters in the j hat direction. Okay? And we can solve for omega bc in this equation. So omega bc equals to negative two root three in the k hat direction, and this is radians per second. Okay, so now we have omega BC, we can solve for VC now. Okay. So we see again, we can use the same equation about the ICS IV, but with a different radius. And with knowing omega BC, so let's do that. So we have we've c is equal to the velocity at the icees, that v plus omega VC, cross product RFC with respect to the IC lead. Okay, so we just applied this equation added this same equation here at a different point, instead of B, we applied it at C here. Hey, and this time we know B, VC, you know that the velocity of the ICS that we always has to be zero, so we can solve for VC. So this is what our VC is equal to. Omega VC is negative two root three in the k hat direction, cross product to RFC with respect to the icy Zed V, which in this case is just vertically up. It's this length over here. So R of C with respect to iceis v. And so it's essentially just in the vertical positive y direction. So root three over six in the j hat direction. And this is again, meters. Okay. And so when we solve for this, we get that VC is equal to one meter per second in the i hat direction. Okay. Now that we know what the C is equal to, we can solve for omega CD. So omega, we can apply the same equation, so we see is equal to V of D, which is the IC is that V because it's the point about which pin so no velocity, which is, that's going to be zero plus omega c, d cross product to R of C with respect to D. Okay, so I essentially apply that equation to the last rod. But here I see that v is at D, because everything is rotating about this point which has a zero velocity. And so we can solve for vc, because we have omega C, D, or we can solve for omega C, D, because we have VC, and we have this radius here. So again, this equation turns into one meter per second in the i hat direction, is equal to omega c, d in the k hat direction, cross product to RFC with respect to D, which is just vertical y direction on 0.2. So process 0.2 meters in the j hat direction. And so that's going to yield omega c, d is equal to negative five radians per second in the k hat direction. Okay. So once we have all of these velocities, we can then solve with the kinetic energies. So now we can do energies. So first of all, we're going to split this off into three sections energy for the, for the three bars, so we have that the total energy. So t total, I'm going to be equal to T of AV plus to BC, plus T of CD. And we need to find each of these. So each of them will be equal to so for example, T of a B will be equal to one half i omega squared. Okay, so this is the definition of kinetic energy. So this is really important. You can find the kinetic, the kinetic energy is always the same for the bar, but the inertia and omega, well, the inertia is different about different points on the bar. Okay? And you have to be kind of really smart about what inertia you use. So that you can use the actual value of omega and then you don't don't have a linear velocity. Okay? So you can have a rod that is translating and rotating. Okay? And if you pick different points on that rod, you're gonna have different combinations of the two, but they always add up. So the linear component and the rotational component, they always add up to the same energy. But if you pick the right point, you do not have both components, so it's easier to calculate. So in this case, for T of a B, I will use is equal to one half, I have a, so I about a omega a b squared. Okay. And why do i do that because I know he has zero velocity. And so I can easily find ay ay ay ay ay, just with a formula given for a slender rod. But then I can use omega a b squared, which I have, and at this point, there's no linear velocity. Okay? So that's why I can eliminate that one half mv squared term, and it's just a one term calculation, less chance of making mistakes. So in this case, I will just Go, plug in the numbers, I have a is going to be equal to 1/3 M L squared. So 1/3 times five kilograms, that's the mass times 0.4 meters squared. That's the idea a, so about the end of that rod, 1/3 Ml squared. And then omega A B has given as five radians per second, on its negative five, but again, five radians per second, on but it's squared, so the negative goes away. So we have that T of Av. kinetic energy of a, b is equal to 10 over three tools. Okay, let's move on to BC, which is a bit more complex. So again, we have the same formula by omega square. Now we have to be smart about which point we take I so we can actually use this equation and there's no linear velocity term. And in this case, it's going to be the IC Zed v again, and so we calculated with the ICS, SV is four bar VC, which is down here. So we have to find I about the ICS at V, not the end. But the IC that V for bar AV the ICS at V was at a so that's why we picked a. So there wasn't that linear velocity term. Here, we have to pick icees. That, so what we do is we find the inertia about the center with 112 Ml squared. And then we move with parallel axis this distance here. And what is that distance? That is essentially the hypotenuse of this triangle here. So it's the hypotenuse of half of this thing here. Okay, what's this thing squared plus this thing squared, and this thing is just half point five, which is 0.25. Okay, so not too complex. So we have one half times by have the ice Zed V, omega VC square, okay, this is going to be equal to actually, I'm going to go on a new line, because it's a bit of a longer version. So one half times I have a CSV, which I said is gonna have two terms. 112 Ml squared, which is five kilograms, times 0.5 meters squared, plus and r squared, choose five kilograms, times R, which actually, let me just move everything back a little bit, which is going to be equal to this. Well, here, R is the square root of this component here. Plus this half component there. But since we need our square, then we can just take not take the square root. So root three over six squared plus 0.25 squared. And remember, the 0.25 comes from half of that point pipeline. Okay. And so this is the inertia, and then omega is negative two root three squared, that negative goes away. So T of bc is equal to 35 over two joules. And then we have T of CD, which is essentially just like the first one is going to be equal to one half i omega squared, one half I d omega c, d squared, which is equal to one half times 1/3 Ml squared, which is the inertia that's five kilograms times 0.2 meters squared times 0.5 radians per second squared, so TFC, d is equal to five over six joules okay. So this is again, meters, meters. So if we add to a b is equal to total is equal to T of a B plus to BC was to C D, G is equal to 10 thirds joules Plus 35 or two joules plus five over six joules. So t total is equal to 21.67 joules. And this is our final answer.