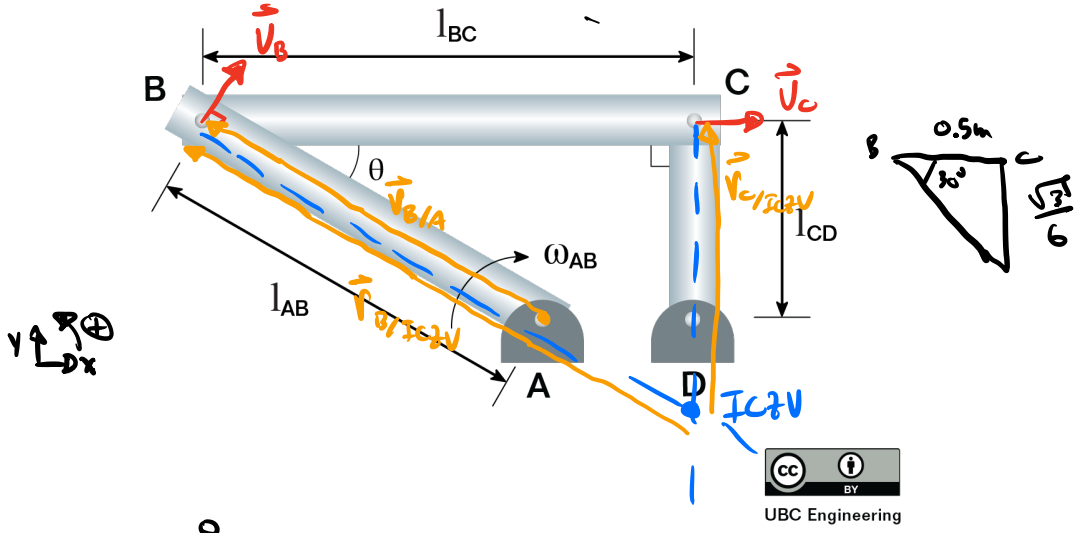
Students are testing a platform mechanism consisting of 3 linkages. If each linkage can be treated as a slender rod, determine the total kinetic energy of the mechanism. Each rod has a mass $m = 5kg$ and the lengths are given as $l_{AB} = 0.4m$, $l_{BC} = 0.5m$, and $l_{CD} = 0.2m$. Rod AB forms an angle $\theta = 30\ deg$ with the horizontal. Rod AB rotates at an angular velocity of $\omega_{AB} = 5\ rad/s$.



$$\vec{V}_B = \vec{V}_A^{\,0} + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$= (-5\ rad/s\ \hat{k}) \times (-0.4\cos 30^\circ\ \hat{i} + 0.4\sin 30^\circ\ \hat{j})$$

$$= 2\cos 30^\circ\ \hat{j} + 2\sin 30^\circ\ \hat{i}\ [m/s]$$

$$\vec{V}_B = \vec{V}_{IC_{2V}}^{\,0} + \vec{\omega}_{BC} \times \vec{r}_{B/IC_{2V}}$$

$$2\sin 30^\circ\ \hat{i} + 2\cos 30^\circ \hat{j} = (\omega_{BC}\ \hat{k}) \times (-0.5\ m\ \hat{i} + \tfrac{\sqrt{3}}{6}\ m\ \hat{j})$$

$$\omega_{BC} = -2\sqrt{3}\ \hat{k}\ [rad/s]$$

$$\vec{V}_C = \vec{V}_{IC_{2V}}^{\,0} + \vec{\omega}_{BC} \times \vec{r}_{C/IC_{2V}}$$

$$= (-2\sqrt{3}\ \hat{k}) \times (\tfrac{\sqrt{3}}{6}\ \hat{j}\ m) = 1\ m/s\ \hat{i} = \vec{V}_C$$

$$\vec{V}_C = \vec{V}_D^{\,0} + \vec{\omega}_{CD} \times \vec{r}_{C/D}$$

$$1\,m/s\,\hat{i} = (\omega_{CD}\,\hat{k}) \times (0.2m\,\hat{j}) \Rightarrow \vec{\omega}_{CD} = -5\,rad/s\,\hat{k}$$

Energies:

$$T_{TOT} = T_{AB} + T_{BC} + T_{CD}$$

$$T_{AB} = \frac{1}{2} I \omega^2 = \frac{1}{2} I_A \omega_{AB}^2 = \frac{1}{2}\left(\frac{1}{3}(5kg)(0.4m)^2\right)\left(-5\frac{rad}{s}\right)^2$$

$$T_{AB} = \frac{10}{3} J$$

$$T_{BC} = \frac{1}{2} I \omega^2 = \frac{1}{2} I_{ICM} \omega_{BC}^2$$

$$= \frac{1}{2}\left(\frac{1}{12}(5kg)(0.5m)^2 + (5kg)\left(\left(\frac{\sqrt{3}}{6}m\right)^2 + 0.25m^2\right)\right)(-2\sqrt{3})^2$$

$$T_{BC} = \frac{35}{2} J$$

$$T_{CD} = \frac{1}{2} I \omega^2 = \frac{1}{2} I_D \omega_{CD}^2 = \frac{1}{2}\left(\frac{1}{3}(5kg)(0.2m)^2\right)\left(0.5\frac{rad}{s}\right)^2$$

$$T_{CD} = \frac{5}{6} J$$

$$T_{TOT} = T_{AB} + T_{BC} + T_{CD} = \frac{10}{3} J + \frac{35}{2} J + \frac{5}{6} J$$

$$\boxed{T_{TOT} = 21.67\,J}$$