

Problem 20-R-WE-DK-6

In this problem, we have a rotating rectangular plate. And we're asked to find the difference in kinetic energy between was rotating around its center of gravity compared to when it's rotating about a point p, which is a distance d away from the center of gravity rotating at the same angular frequency of three radians per second. So in this case, we're just going to split it off into two different scenarios, and calculate the energies, the kinetic energies, and then subtract the two to find the difference in kinetic energy. So we're going to look at first the case about which it's spinning about the center of gravity. So case one is spinning about G. Okay, so about g, we know that the kinetic energy or also known as T_G , is going to be equal to one half $I_G \omega^2$. Okay, so we need to find what it is. So I_G is essentially the I for a plate, which is given by the dimensions AB about g, which for a plate like this is given by the following formula $\frac{1}{12} m (a^2 + b^2)$. Okay, so if we actually plug in values into this equation, we get that $\frac{1}{12}$ times the mass which is 14 kilograms, times four meters squared, plus six point or three meters squared gives us we can plug this in, and it gives us that I_G is equal to 29.2 kilograms meters squared, then we can plug this I_G into this equation to solve for T_G we know ω , so we can solve for T_G , T_G is equal to one half times 29.2 kilograms, meters squared times three radians per second, and all squared and T_G is 131. point two five joules. Okay. So this is case number one. So spinning about G. case number two is spinning about P. So in this case, we still have the energy, so T_P being equal to one half $I_P \omega^2$. Okay. But I_P is different than I_G . So this is I about G. This here is I about this point p, so everything spinning about this fixed point P. So we have to calculate I_P . Now how do we do that? Well, we just use parallel axis theorem. So we already have I_G , and then we are going to add that parallel axis term to move away from G to a point P. So I_P is going to be equal to the same thing we had before for I_G , so $\frac{1}{12} M (a^2 + b^2) + m d^2$, where this D here is this distance here are called D on the diagram, the distance between G and P. Okay, and that extra term is the parallel axis term. So if we plug everything in, we get the following. One over 12 times 14 kilograms, times four meters squared plus three meters squared plus 14 kilograms, times 6.5 meters squared. Okay, and so I_P is equal to 1862 divided by three kilograms, meters squared is also equal to 620.7 kilograms, meters squared. Okay, now we can again do the same thing we did before, take this number here, which is I_P , plug it into here, we know ω , so we can actually find T_P . So T_P is going to be equal to one half times I_P , which is 620.7 kilograms, meters squared times five radians per second squared, and this yields to 793.5 joules. Okay, this is much, much more kinetic energy than spinning about the center of gravity. Okay, so now if we want to find ΔT , that's going to be equal to T_P minus T_G , the absolute value of it because we don't really care for the sign. So if we do this, we're going to get 793.5 joules minus 131.25 joules, which equals two and this is the absolute value of it, which equals to 662.25 joules. That's our ΔT or difference in kinetic energy between the two cases.