Problem **20-R-KIN-DK-33**

In this problem, we have a cart that's holding a plate, the cart is given an acceleration. And we're asked to find what is the acceleration and the forces reaction forces at p from this acceleration of the system. So the first thing we do is we draw a freebody diagram of the plate. And then we're going to analyze the plate. So this plate we know is going to have a force due to gravity, and then there's going to be a reaction forces over here. And then the whole system is also going to have an angular acceleration due to that acceleration a over here. So let's draw a freebody diagram neatly. So this is our plate Android a bit bigger. And the center of gravity is located at the center here. So we're going to draw first for us, which is the force due to gravity, which points downwards, and we're gonna call this f g. And then we have our reaction forces of point P. So we're going to call these aX and aY, ax and ay, and I just assumed the direction. And if we get a negative number, they'll be in the opposite direction. And then for the kinetic diagram, we're going to have an acceleration of the cart, which directly transfers to a linear acceleration of point p, and then an angular acceleration of the system, alpha. Okay, and I'm also going to draw in my reference system to the right here. So to the left, x is positive to the right, y is positive, no rotation is positive in the counterclockwise direction. So the first thing we do is we are going to, we're going to do a sum of forces and x, the y and then a sum of moments, just to get the three first three equations. So first of all, we're going to take a sum of forces in the x direction, which is going to be equal to m times Ag of x. I also forgot to mention at G there's an acceleration, agx, and then there's an acceleration in the y agy. And again, these directions are assuming that they're up into the right, if we get a negative number, it's going to be in the opposite direction. Okay, so ag x equals to a x, because that is the only force in the x direction, okay, then we have our sum of forces in the y direction, which is equal to m times a G y, which if we implement this, we get ay minus fg is equal to mAgy. Like that, well complete. Okay, so these are 2 force balances, then we're gonna move on to a moment balance, and we're going to do a moment balance about the center of gravity, G, which is equal to Igy alpha. So if we implement this we get on the two forces that create moments are a x and a y. So we're gonna get ax negative aX, because it makes everything spin clockwise and our positive direction is counterclockwise, times 0.8 meters. Because that is this distance here, which is essentially y, and y is two meters. And then we have minus ay times 0.4 meters. And since this, this distance here is one, and this distance here is 0.6. The remaining distance between here and here is 0.4 meters. It's negative again, because it makes it spin clockwise and our positive is counterclockwise. This is equal to five G, alpha. And we need to calculate a G, so I'm just going to do it on the side here. IG is going to be equal to one 1/12 times m times L squared, which is the length of the plate times the width of the plate squared. So this is going to be 112 times five kilograms times two squared plus 1.6 squared, because this distance here is 1.6, because y is 0.8. Okay, and once we solve this, we got that i g is equal to 41 over 15 kilograms, meters squared. Okay, so we have Ig. And as you can see, we have three equations, but we have five unknown, so ax Ay, agx, agy, and then we have alpha. So we need two more equations. But fortunately, we can actually add some equations from the acceleration equation. So we know the acceleration at point p, and that's going to constrain our system and add the necessary equations to solve for all of the unknowns, okay, since we know the acceleration at point p, we can relate the acceleration at point G two point p, okay? So that acceleration equation is the acceleration at G is going to be equal to the acceleration at p plus alpha cross r, g with respect to p minus omega squared times R of g with respect to P. Okay, now there's no omega initially, so we can actually cancel out this omega term here, you can set this to zero. And we're left with that equation there. ap is known, we're given that that is, that's given. And we need to solve for the remaining terms. So we don't know AG, that's what we're trying to find. So we will do the following, we will solve we will solve for RG with respect to P, and plug everything into the equation. So ag or actually, let me plug in ag. So ag is actually ag x any i hat direction, plus a G y and the j hat direction, it's going to be equal to A p and a P is going to be a linear acceleration, because P is attached to the cart of two meters per second squared. So this is going to be two meters per second squared in the i hat direction. In the positive I have direction, because it is to the right. Okay. And then we have alpha cross cross r, so we don't know alpha, we'll just leave it at that. But we'll give it a direction that is in the k hat direction. And this is going to be cross product to rg with respect to P and RG with respect to P we're we know this, it's going to be 0.4 in the i hat direction, minus 0.8. In the j hat direction, and that's because that radius, I'll draw it in orange over here, start goes like that rg with respect to P, so we have a negative 0.8 component in this direction, that's just y. And in the x direction, we're given we need this length here positive because it goes to the right, and that's 0.4, just like I described before. So we can actually compute this cross product k with I and then k with J negative J. And we can actually solve for two equations, two unknowns, or two equations here, we're gonna have the i hat equation and the j hat component equation. So let's simplify that into one equation and then split it off into two. So we have a sorry, let me go back to black. A g x in the I had direction plus agy in the j hat direction is equal to two meters per second squared in the I had direction plus my 0.4 Alpha in the j hat direction, plus 0.8 Alpha in the i hat direction. Okay. Now let's split up the i and j components. So we have a g x equals to two meters per second squared plus 0.8 alpha. And we have a G y is equal to 0.4 alpha. Okay, so given this, let me make this look like a better alpha. Given this, we can now we have now added two equations, and we've added zero unknowns. So we have five equations and five unknowns, and we can solve this whole system. So to solve we get the following equations. And once we solve this system of five equations, five unknowns, we get the following answers. So a x is going to be equal to x is equal to negative 6.4 Newtons, a y is going to be equal to 40.8 Newtons and alpha is equal to negative 4.1 radians per second squared. And additionally, we can also solve for Agx Agx, that is going to be equal to negative 1.28 meters per second squared and aGy is equal to negative 1.64 meters per second squared. And these are our final answers.