Problem **20-R-KIN-DK-31**

In this question, we have a yoyo spinning at a given omega with a given tension. And we're not sure if it's either slipping or rolling without slipping on the ground. So we have to test that. But we're asked to find what the angular acceleration and linear acceleration are of the yoyo. We're also given all the friction coefficients and the angle, theta. So as I mentioned, there's two cases for this question, which we don't know which one is occurring, either slipping or no slipping at the bottom here, where the yoyo contacts the ground. So we have, we're given the angle theta, and we have the following forces. So we have a tension for us that's pulling g up along that line with the beta angle. So that's that we're going to call T tension T, we also have the weight of the Yoyo creating g downwards. And then we have two more forces the normal force and the friction force at the contact point here, so the normal force points up, and the friction force is going to point towards the right, and we're going to call this f of f, and the direction of the friction force is given by this direction of omega. So since this is slipping this way, the friction force counteracts that direction. So that's why it's pointing towards the right, so we have four forces. Now, with this, we can take our sum of forces an x and y direction, and then take our sum of moments. So I'm actually going to draw on the coordinate system over here, x is towards the right, y is upwards, and rotation is positive counterclockwise. So given these forces, we're going to do our sum of forces X, Y, and Z are sum of moments. So sum of force in the x direction is going to be equal to m times ax, I also forgot to do the kinetic diagram. So we have an acceleration only in the x direction know why white no acceleration in the y direction because it doesn't detach. And we also have alpha, which I'm going to assume is going in this direction, but could be going in the opposite direction. So this ax is just simply going to be a because there's no acceleration in the y direction, okay. And if we implement this, we get force of friction minus tension times the cosine of theta is going to be equal two times eight. Okay, then we can do some a force in the y direction, and this is going to be equal to zero, because a y is zero. So this equation yields t times sine of theta minus F, G, which is the gravitational force plus n is equal to zero. And then we have the sum of moments. And we're going to take the point G, because of convenience. So if we take the point G here, on this T force, which is slanted, an angle will goes away. F, G, and n all go away, so we only have f of f. So it's much simpler to do that calculation. And this is going to be equal to Ig alpha. So we get the following. f of f turns the radius of the yoyo is equal to Ig alpha. Okay, and we do not know it, but we are given the radius of gyration. So we can calculate it based on the radius of variation. So, on this side here, my G is equal to mkg squared. And this is equal to 0.2 kilograms times 0.02 meters squared, which is equal to 0.00008 kilograms meters squared. And this is going to be Igy. Okay, so we actually do have it. So given this sum of moments, we can now test the two cases. So we either have rolling without slipping, or we have rolling with slipping. And the difference between the two is that there's two different friction coefficients in each case, so we have to pick which one works best. And when we test, if our hypothesis doesn't match the results we get, then obviously, that case is impossible. And we have to either test the other one, or ensure that the other one works. Okay, so we're going to first test rolling without slipping. So case one, rolling without slipping. So what we have to enforce for this case, is that since there's no slipping, we have that the force of friction must be lower than or equal to us times and mu s being static friction coefficients, because there's no slipping, so it's static, okay. And this ensures that this friction force here is created by this normal force, and it has to be a lower than or maximum at max equal to the normal force times that static friction coefficient. Okay. So once that is ensured, then we can if that holds, then we have rolling without slipping. And to test the test this, we're just going to plug in the numbers on and see what we get. Okay. So, we can relate alpha and ag with the following equation A G. So this is essentially a, aG in the x direction, which is essentially a is equal to alpha cross r. Right. And if we solve this, we get, since we know R, because r is 0.03

meters, we get negative 0.03 Alpha in the i hat direction. Okay. So a has to be negative 0.03 alpha and I have direction and this is because this point, we're relating this point here, which doesn't have any linear acceleration to the linear acceleration a of this point here. Okay. So this is essentially relating A, which is a gx to alpha. And we're, again, it's going to be in the negative I had direction with respect to alpha. Alright, so once we have that, we can actually plug everything into the equations and solve because for this case, again, we're given T and we're given this theta angle, and we can solve for all of the other unknowns. So let's do that. This equation here, we can solve for F, while relates f of f and alpha. So it's still not the full equation. But in this equation here, we can directly solve for N. So we can plug that in. So N is going to be equal to negative 0.4 Newtons times sine of 60 degrees plus 0.2 kilograms, times 9.81 meters per second squared is equal to 1.62 Newton's for that N. Now, once we have N, and then we have this equation relating alpha and f of f, we can solve for f of f from this equation here. f of f is equal to 0.00266 alpha, I just simplified everything. So we have we solved for n fully. And we solve for f of f in terms of alpha. Okay. And then from our third equation, we can relate f of f and alpha together. Okay. All right, so we're going to plug one into each other, to actually solve for the force of friction. So with our last equation, we have f of f times R, which is 0.03 meters, is going to be equal to 0.00008 kilograms meter square times alpha. And so when we combine this someone call number one, I'm going to call number two, one, and two, we get the following. f of f is equal to 23.08 Newtons. And we've already solved for N. So now we can test this hypothesis over here, because we have us and that's equal to 0.3. So if we test that we get so which is equal to we said 23.08 Newton's is going to be smaller than or equal as we need it to be new s, which is 0.3 times 1.62 Newton's okay. And if we get this, we get that. If we test this, we get that this does not hold. This does not work. It's not smaller than it's actually bigger than So, um, this concludes that we cannot have rolling without slipping there must be slipping. Okay, so that leads us to case number two. Rolling with slipping and I'll put a star next to the cases. Okay, so rolling with slipping here, instead of using the static friction coefficient, you can use the kinetic friction coefficient. And here, what holds is that the force of friction is equal to mu k times and Okay, so we can directly plug this back into these equations over here. And this force of friction goes away. And this force of friction goes away. We're solving for alpha, normal force and the acceleration, three equations, three unknowns, we can directly solve for everything and get values. So this is what we get 0.4 Newton's times sine of 60 degrees minus 0.2. This is the mass kilograms times 9.81 meters per second squared plus n is going to be equal to zero. so here we can directly solve for n, which is equal to 1.62 Newtons Okay. Then we have our second equation, or the next equations 0.00008 alpha is equal to 1.62 Newton's which is our normal force times 0.2 kilograms times 0.03 meters. and here we can directly solve for alpha is equal to one to 1.2 radians per second square. And lastly, we can solve for the linear acceleration a. So we have 0.2 kilograms times a is equal to 0.2 kilograms times 1.62 Newtons minus 0.4 Newton's times cosine of 60 degrees. And here we solve for A, which is equal to 0.61 meters per second squared. So our final answer is our alpha is equal to one to 1.2 radians per second per second square, and a is equal to 0.62 meters per second square and these are our final answers.