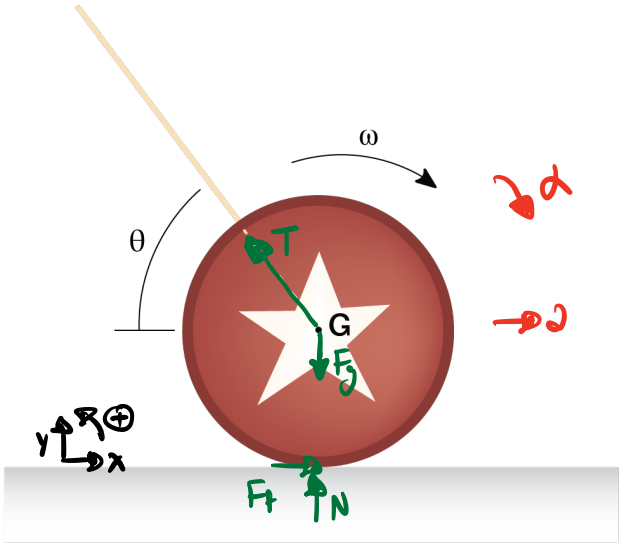
A mechanical engineering student has been practising his yoyo tricks because he has too much free time. For one trick, he spins the yoyo such that it contacts the ground and moves forward, emulating someone walking their dog. If the yoyo has a radius of gyration $k_G = 0.02m$ and a mass of $m = 0.2kg$, determine the acceleration and angular acceleration of the yoyo when the tension in the string is found to be $T = 0.4N$. Assume the string is at its full extent and does not roll up as the yoyo rolls. Assume there is also no friction where the string slips around the yoyo's inner axle. The coefficient of static and kinetic friction are found to be $\mu_s = 0.3$ and $\mu_K = 0.2$ respectively. The angle is $\theta = 60\ deg$ and the radius of the yoyo is $r = 0.03m$. Take the initial angular velocity of the yoyo to be $\omega = 6\ rad/sec$.



$\sum F_x = m a_x \Rightarrow F_f - T\cos\theta = ma$

$\sum F_y = 0 \Rightarrow T\sin\theta - F_g + N = 0$

$\sum M_G = I_G \alpha \Rightarrow F_f r = I_G \alpha$

$I_G = mK_G^2$

$= (0.2kg)(0.02m)^2$

$I_G = 0.00008\ kgm^2$

★ Case #1 Rolling without slipping

↳ $F_f \leq \mu_s N$

$\vec{a}_{Gx} = \vec{a} = \vec{\alpha} \times \vec{r} = -0.03\alpha\,\hat{i}$

$(0.2\,kg)(-0.03\alpha) = F_f - 0.4N\cos 60^\circ \Rightarrow F_f = 0.0026\alpha$ (1)

$N = (-0.4\,N)\sin 60^\circ + (0.2kg)(9.81 m/s^2) = 1.62\,N$

$F_f(0.03m) = (0.00008\,kg\,m^2)\alpha$ (2)

(1) & (2) $\Rightarrow$ $F_f = 23.08\,N$

Test: $F_f = 23.8NN \not\leq (0.3)(1.62N)$ does not work

$\therefore$ there must be slipping

☆ Case #2 Rolling with Slipping

$\therefore F_f = \mu_k N$

$(0.4\,N)\sin 60^\circ - (0.2\,kg)(9.81 m/s^2) + N = 0 \Rightarrow N = 1.62N$

$(0.00008\,kg\,m^2)\alpha = (1.62N)(0.2kg)(0.03m) \Rightarrow \alpha = 121.2\ rad/s^2$

$(0.2kg)a = (0.2)(1.62N) - (0.4N)\cos 60^\circ \Rightarrow a = 0.62\ m/s^2$

$\alpha = 121.2\ rad/s^2$

$a = 0.62\ m/s^2$