Problem **20-R-KIN-DK-28**

In this question, we're dragging a modern art sculpture by a rug. And we're asked whether the sculpture is going to tip or slip. First. We're given all the coefficients of friction, static and kinetic, the mass, and the radius of gyration. So first of all, we're going to analyze the two cases. case one is if we have slipping, in case two is tipping. So slipping is a simpler case. And then we have tipping, which is a bit more complicated. So analyze each in detail. And we're going to draw a freebody diagram for each. But essentially, always start with a diagram, we have our forces. So actually, I'll draw forces in green, we have our weight, this is fg. And then we're going to have a normal force N and we're going to have our friction force, and our friction force is going to point towards the right, force of friction, okay, and this is because we're expecting an acceleration of the rug, a little drawing right here, as a, and this block is going to start slipping, it's going to accelerate with the rug. But if it slips, it's going to move in this direction. So force of friction is going to act in the opposite direction. So that's why it's pointed towards the right here. Again, this force of friction and this normal force are applied on the bottom surface, but we haven't defined yet, at which point. So here, I've just drawn them arbitrarily in the center, but it's not going to be the center for each case. Okay, so for slipping, it doesn't matter where that friction force and that normal force act. So I'm not going to draw the whole structure like this complicated shape, I'm just going to draw a rectangle for a free body diagram, and then I will add stuff to it. Alright, so in our case, I'm going to draw G is going to be over here. This is G, the center of gravity that's identified there, it's not Central. So in the case of slipping, so we're gonna have our acceleration always going this way. Right? So this is going to be A and in the case of slipping, what happens is the force the friction force, is not enough to keep the block stationary, right? And so the block starts slipping backwards. Okay. So essentially, it doesn't really matter at which location, this force is applied, as you'll see later from our moment diagram, but essentially, we're going to have our gravitational force pointing down of G. And then we have a normal force pointing upwards and a friction force pointing towards the right. Okay. And in this case, alpha, the angular acceleration will be zero, because this is not tipping. This is slipping. So we don't have an angular acceleration in this case. And the condition for slipping is that, to avoid slipping is that the force of friction, So f of f needs to be lower or equal to mu s times N so again, static friction coefficient, because we are talking before slipping, not after slipping so it's static. And right at the point where it starts to slip, the equality occurs. Okay, so when F of f is equal to me as n, that's the maximum point before then it starts slipping. Okay, so we're going to be applying this condition here to this system to solve for all of the unknowns. Okay. So, this is the slipping case. Pretty simple. The Tipping case is a bit more complex. So we have to look again, we still have our force of gravity here at G, FG. But the location of our forces the normal and friction forces matters in this case. Because when something tips, the forces will always be at the outermost edge. So the normal force again will be right at the edge, same thing as the friction force. And this is because once the force goes outside on then it, the body would have to start to make right in the force has to be applied at a point on the body. And the last point it can be applied to is to the edge for it to pivot around that and then start tipping. So, again, we have our same acceleration, but in this case, we also have another part of the kinetic diagram, which is the alpha component. So we have an alpha directed in this direction, because everything is going to start tipping. And so we it's not going to be fixed. And again, the acceleration is only an extraction for both cases, which is going to simplify our blob problem, I'm also going to draw in the coordinates for both systems, I'm just going to draw them once on x, y positive rotation counterclockwise. Okay. So now, the condition for this system. And then I also forgot to write the other condition here, which is alpha equals to zero. The condition for the system to the right, is that these forces are on that outermost edge. And that's going to ensure we have tipping and then we also have an alpha, which ensures everything is rotating. Okay. So let's solve this question. Let's solve for these two systems, doing some forces and moments and see what we get. So in the end, we're trying to solve for this acceleration here, right? That's what the question asks. We're given this fg, we're given this static friction coefficient. And we have to essentially solve for everything else. So let's start with case one. So again, I'm going to keep it divided between the two. So case one, we have our sum of forces in the x direction, which is going to be equal to max, which is just ma. So this is going to be equal to f of f equals to ma. Alright, so A is the acceleration in the x direction, then we have our sum of forces in the y direction, which is equal to zero because there's no acceleration in the y direction. And when we implement this, we get the following. And minus mg is equal to zero. Okay. And so here with this equation, we can directly solve for N, right, because we have mg. so we can directly solve for n. And then we have our last equation, which was this constraint here, where f of f has to be equal to mu s times N so since we can solve for N, we can plug in N here because we have us and we can find the force of friction. Once we have the force of friction, we can plug it in here and we can solve for the acceleration. Okay? So once you plug all these numbers in, you get the following. And this is the final acceleration a, for the case of slipping. Okay, so this is I'm going to underline it in red. This is what we're gonna we're gonna need. So this is essentially this exploration we found we need on the carpet for this living case. So this is the maximum acceleration before tipping occurs. On now let's analyze tipping. So if in the tipping case we got a lower acceleration, then that means tipping will occur before slipping. And the opposite is true if we get the opposite results, okay, so we're essentially solving for the acceleration in each case, and then comparing the two accelerations. So let's do a sum of forces for the second case. So some force in the x direction yields the same exact equation. So equals To me x, therefore, f of f is equal to a, and then a sum of force in the y direction also use the same equation. So we get n minus mg is equal to zero. Okay. But now, in this case, we don't have this constraint here. So we need an extra equation, or else we cancel for all of these unknowns, a and f of f, right, and then we also have alpha. So we need to introduce an extra equation, which is our sum of moments, which we didn't have to do for the other case, but we do need over here. So this is the sum of moments. And we'll take this about the center of gravity, G. Okay. So this is going to be equal to I alpha Ig alpha. So once we implement this, we get the force of friction times 1.6 meters, minus N times 0.1 meters, is equal to Ig alpha. Okay, so we have done our moment balance, and we have four unknowns and three equations here. So 1234. But we know that right at the moment that this starts tipping alpha is actually zero, right at that moment, before tipping, when it starts, tipping alpha is going to start is going to have a value. But right before it's not alpha is going to be zero. So we can actually get rid of this term, because we're looking at the point right before tipping, not at tipping. So that's why we set this alpha equals to zero. In this case, alpha is always zero, because it's going to slip, it's never going to start rotating. In this case, alpha is not always zero, but right before it is zero. So that's why we eliminate that. So in this case, now, we can actually solve for our, our system of three equations, three unknowns, so just like we did before, and solve for n, we can plug in and over here and solve for f of f, we can plug in f of f in here to get a. So let's do that, we get that we get the following to 18 kilograms, times 9.81 meters per second squared, times 0.1 meters divided by 1.5 meters. And f of f is going to be equal to 32.32 Newton's and given this f of f. So this is essentially plug in these two equations together to get this. Then we take this f of f, we plug it into here, so we divided by 80 kilograms to get the acceleration. So A is equal to 32.32 Newtons divided by 80 kilograms, which is equal to 0.65 meters per second squared. Okay, and so this is the other value of acceleration. And as you can see, this value here is smaller. So let's write that out. If that's smaller, and that means it's going to occur before the other one, okay. So, since a, in this case it was tipping is smaller than a. Therefore, the system will tip before slipping. And the magnitude of the acceleration is equal to 0.65 meters per second squared.