Problem **20-R-KIN-DK-21**

In this problem, we have three slender rods that are welded together. And they're attached at a, rods, AB, and CD start off horizontal. Meanwhile, on rod BC is an angle theta with respect to rod AB, we're asked to find the initial angular acceleration alpha of the system after it is let go from rest. So looking at this diagram, we can see that everything is pinned at A and is rigidly attached, and we're going to get an alpha, that's going to point in this direction for the whole system. Now, the forces on this body are as follows, there's going to be a gravitational force downwards for each of the three slender rods. And then we have reaction forces at A. Okay, so this is an mAB, G, and mBC, G, mCD, G. Okay, so we're asked to find what this initial alpha is. And, as always, we're gonna start with our freebody diagram. So we're gonna detach everything and redraw the whole system with our forces. So we have first rod, second rod, and the third rod. And we have the gravitational force for each of these rods acting in the negative y direction. And it acts at the center of the rods, because we're assuming a constant density. So again, this is m, a BG, this is an BC, G, and this is C, D, G. And then here, we have our reaction forces, a Y and a x. And, or, again, our coordinate system is x along towards the right, y in the vertical direction in a positive rotation, be counterclockwise. Okay. And for our kinetic diagram, we're going to draw in our alpha, which is our angular acceleration. Okay, so this is the diagram. Now we can start solve for alpha with our force balance and our movement balance. So if we do a force balance, all we get is we can get to the reaction forces that a but we're not asked for these reaction forces, we're asked for alpha, the only way we can get alpha is with some of the moments. So that's what we're going to use, we're going to send the moments. And to eliminate this reaction for us from our equations. Since those are unknowns, we will simply take the sum of moments about A, okay? So that will eliminate these forces because they don't have moment arms. So we're going to take the sum of moments about point A, and this is going to be equal to I alpha, and this is specifically I about eight. Okay, so, first of all, let's calculate this I. So, each of these three rods will have a different eye, about eight. So we're going to calculate each separate and then we're going to superimpose and add them all together to find the total Ia. Okay, so we're going to call this rods, AB. So this is rod AB, this is rod BC, and this is rod CD. Okay, and so we're gonna do each separately. So I have AB, it's going to be equal to 1/12 m l squared plus md squared. Okay, so this is going to play for each of the rods, but l and D change. So all the rods have the same mass 2 kilograms, but l is the length of the rod, D, this is the parallel axis portion is to, since this here will give you and it's 1/12. Sorry, this here will give you the I at the center of gravity, so at the center of the rod, but we needed to be at the end of the rod, or we're going to shift this I by a distance d. So for this case, we have to for the first rod, AV, we have to shift it to the right, by half my length. For ride BC, we have to shift it by this length for rod CD, we have to shift it by that length, Okay, so this D here is the distance between the center of gravity of the rod on 2.8. Okay, l here is just the length of the rod. So let's do it for the first one. So we have 1/12 times two kilograms, times L squared, which is 1.5 meters squared, okay, plus the mass, which is two kilograms, again, times, here we have half the length, so 0.75 meters squared. And this is because this 0.75 is half of this distance, because we're moving from here to point A. Okay, so that's I have a be at a, okay. Now let's do I have BC, again, with respect to eight. So this is again, 1/12 mL squared plus d squared. And again, we have the same maths, because they all weighed two kilograms. But now this distance here, and this length and distance are going to be different. So this here is one meter square plus mass is the same. But now we have to do some trigonometry, because this length here is the distance between CG here to a. So if we redraw that triangle, we have this, which is 1.5 meters, and we have that and we're interested in this length over here. Okay, and we know this angle, this angle here is theta, which is 45 degrees, okay. And then we also know that this here is 0.5 meters, okay. And we are interested in finding this here, which is D, okay. And if you solve for D, with basic trigonometry, you get the D is equal to 1.1997 meters. Okay? So this is going to be 1.1997 meters squared. Okay. And then lastly, we have I of CD, which is again follows the same formula, Ml squared plus d squared. But we're gonna have again, different lights. So here, the length of this rod here is 0.5 meters squared, plus same mass. But now we have an even more complex geometric problem. So for this, we know that this length here is 1.5 meters, then this goes down, and this is like 1 45 degrees, and then it goes that way, 0.25 meters, and we need to find this distance here, which is D. Okay, so let me actually highlight and write of which distance we're looking at. Right. So again, we do some trigonometry. So the way we solve for this is essentially we take 1.5 and we subtract this distance here. And since we know this angle, with cosine, inside our sine, we can find our with cosine we can find this length, so we take 1.5 subtract this length, and we're here and we subtract another 0.25. And we get the vert the horizontal distance that this travels and then The vertical distance is just the sign, and then we square the two and then square root the sum of the two squares to get deep. I'm not going to do it, or else it would be too long. But it's simple trigonometry. And this yields that d is going to be equal to 0.8. Let me move it a little bit. 0.8915. Okay, so 0.8915 meters squared. Okay, so again, it's the magnitude of this, if you split it into the X and Y component, then find the two and then sum the squares. Pretty simple. So once we get these three terms, we can solve for these each. And then we add them up and we get the total ay ay ay. Okay. So I'm going to do that. I'm not going to add it up, I'm just going to give you the final answer. So I A is equal to I have a B plus I have bc plus ilcd, which is equal to 11.2758 kilograms, meters squared. Okay. So this is I know we back to our equation, here we have a, we're trying to solve for alpha. So now we just need the sum of the moments. And so the way we summer moments is we go back to our freebody diagram, and we look at all the moments caused by each grab gravitational force, we find that radius, and we multiply the two to get the full sum of moments. So going back to the sum of moments, about a is equal to i alpha, alpha. implementing it, we get, so I'm not going to write a out yet, I'm going to start with a sum of moments. So we get that the force of the first so I'm going to start from a B here, I'm a big times radius, which is half the length 0.75 meters. And since this is going to get the whole system to rotate this way, counterclockwise is going to be positive. And again, half of the radius. Since this is all perpendicular, I can just divide 1.5 by two and that's why I get 0.75. Okay, then I am going to do the next force, which is m bc G. And again, I add it because it twists everything clockwise, counterclockwise, B, C, G, and this radius here is a bit more complex. Um, so this here is going to be the distance between here this horizontal distance. So again, what I would do is I take 1.5, which is this length, and then subtract this little x component here, which you can simply get by taking the since we know this angle here, we take the cosine of this, this angle, times this length here, which is half of this whole length here, and we subtract it from this whole 1.5. Okay, so I'm going to write that in 1.5 minus 0.5 cos of 45 degrees. Okay. So again, this would give you that distance, that moment arm from A to the location of BC, but it has to be this horizontal distance because this force is fully vertical. And then we are going to move on to the last force, which is MC D, G, and I'm going to go on new line, and C, D, G, and again, this one will be more complex, because we have to get this radius. This distance here, so between here and here, the horizontal distance, okay. And so we're going to follow the same process, we're going to take the this length, subtract this length, alright, subtract this length over here, between here and here. Answer that's again, the cosine of 45 times this whole length here, which is one, and then we're going to subtract to the right to subtract this length here to half of this length to get to this point here, so only half, okay? So this is why this is going to be 1.5 minus 0.5 coasts, or sorry, not 0.5 is going to be one times coasts of 45 degrees minus 0.25. This is all in meters. Okay. So remember, this is the one on the top is minus the full length minus the full length between here and here, which is one is that as close to 45 minus half of this length here, we get to that point there. And then we can take this and equate it to i alpha. And so I A alpha and we have a here, it's just a number that we just solve for. And we also have a G, and we have all of the masses. So here, we can directly solve for this alpha term over here. So, now I'm going to plug everything in and solve for the final answer. Yeah, two kilograms times 9.81 meters per second squared, times 0.75 meters, plus two kilograms, times 9.81 meters per second squared, times 1.5 meters minus 0.5 meters coasts of 45 degrees, dot dot dot plus two kilograms, times 9.81 meters per second squared times 1.5 meters minus one meter cosine of 45 degrees, and is 0.25 meters is equal to 11.2758 kilograms, meters squared times alpha. and So solving for alpha, we get that alpha is equal to 4.24 radians per second squared.