

Problem 20-R-KIN-DK-26

In this problem, we have a wheel that is rotating, and we are asked to find first, given the conditions, the reaction forces of link AB, and the normal force that's acting between the wheel and the ground. And then we're asked to find, given an initial angular velocity, what is their time required to come, for the wheel to come to a complete stop, we're given the coefficient of friction and the geometry along with the mass. And we're also given the radius of gyration of the wheel. So like always, you have to start with a free body diagram. So we're going to start with the wheel. And this is going to have its weight at the center. This is going to be G , then we have our normal force at the bottom. So we're going to call N , and we're going to draw in ω , which is going in this direction. So if ω is going in that direction, then our force of friction must be going in this direction. Okay, opposite to ω at this point here, and then we have the force the reaction force of the link, which is going to be along link AB, we're going to call that F_{AB} , okay? And this is going to be at an angle θ . So again, positive x to the right, positive y upwards, positive rotation counterclockwise. Okay, so this is our freebody diagram of the wheel. So from this, we can remember this force is going to be a long link AB, because nkb can only carry axial forces. So there can't be a force perpendicular, so it's got to be parallel to that. So we're given the direction of that force. Okay, so let's start with the sum forces in the x direction, this is going to be equal to zero, because there's going to be no acceleration of the center of gravity of that wheel. Okay, so we're gonna get the following. And this is going to be equal to zero, so we're taking the x component of this force, and we're taking the force of friction, because these two forces are perpendicular or vertical, so don't act in the x direction, then we can take a sum of forces in the y direction, and again, same thing, no acceleration. And we get the following. So $F_{AB} \sin \theta$ is going to be positive minus mg , plus and equals zero. And then we are going to have the sum of moments. And again, for some moments, we can do it about any point. But we're going to take B because B is the point that has most of the forces going through it. So we can cancel out F_A , B , F , G , and because they have no radius, and so that's going to simplify our equation a lot, also removing the force with an angle, so we don't have to do complicated cross product. And this is going to be equal to $I \alpha$, we do have an angular acceleration in this case. So what this yields is $F_{AB} \cos \theta$ times radius, because $F_{AB} \cos \theta$, we need this distance here, that's the radius of the circle of the wheel, sorry, that's going to be equal to r , α . Okay, so now we can start plugging stuff in and solving for these equations. So the first thing is we need to find I_B , which is easy to find because we have the radius of gyration. So I_B is m times the radius of gyration squared. So that's going to be equal to five kilograms times 0.2 meters squared, which is equal to 0.2. We have I_B and we can solve we have the following unknowns, α force of friction, and F_{AB} along with M . So we need one more equation. And that is, so this is equation one, equation two, equation three, and we have equation four, which is the force of friction is equal to μ_k times the normal force. So now we have four equations, four unknowns, and we can solve the system of equations. And it yields the following. F_{AB} equals to 18.4, Newton's, n is equal to 39.8 Newtons, and then we have F_f is equal to 0.4 Newtons. And α is equal to 11.96 radians per second square. So these first two over here are part of the final answer. Because these are the values for the reaction forces at the linkage, and at the bottom, but we're not done yet. We also are given an initial velocity or angular velocity. And we're asked to find given this friction coefficient and given this geometry, how long does it take to stop? Okay, so we've calculated α , which we will need for this calculation, because we know that we have an initial velocity that ω is going this way. And as time goes on this friction will cause a deceleration. So ω goes this way. But α is actually in the opposite direction, which is slowing down the wheel. Okay. So we know that the formula for to relate ω and α in time, obviously is $\omega = \omega_0 + \alpha t$, the final velocity is equal to the initial velocity plus α times the time, okay, and we know that

omega is going to be equal to zero radians per second, because that's the final velocity, we want it to stop. We have omega not omega naught is equal to 30 radians per second, we're given this in the question. And we know that alpha is 11.96 radians per second squared. So in this equation, we can essentially just solve for time to know we can plug everything into this equation, we have zero radians per second, is equal to 30 radians per second, plus alpha, which is negative 11.96 radians per second squared times t. And remember, we need this negative here, because Alpha and Omega are in opposite direction. so omega is going in the clockwise direction, alpha is counterclockwise. Okay? So again, it doesn't really matter if this is negative, or if this is negative, but one of them needs to be negative, okay? Or else you would end up with a negative time, which is not possible. Okay. So if we solve this, we solve for t and the t is equal to 2.5 seconds. And so this is the time required for the angular velocity to go from 30 to zero radians per second.