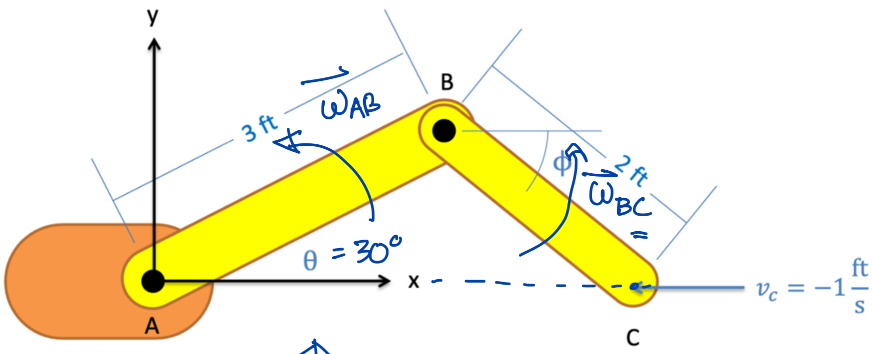
The robotic arm from the previous problem is in the configuration shown below. Assume that theta is currently 30 degrees and that point C currently lies along the x axis. If we want the end effector at C to travel 1 ft/s in the negative x direction, what should the angular velocities be at joints A and B?

Find $\vec{\omega}_{AB}$ & $\vec{\omega}_{BC}$



Known:

$\vec{V}_A = 0$

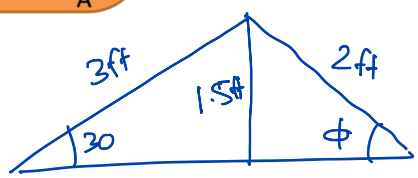
$\vec{V}_C = -1\ ft/s\ \hat{i}$

$\vec{r}_{B/A} = 3(\cos 30\hat{i} + \sin 30\hat{j})\ ft$

Assume positive

$\vec{\omega}_{AB} = \omega_{AB}\hat{k}$

$\vec{\omega}_{BC} = \omega_{BC}\hat{k}$



$2\ ft\ \sin\phi = 1.5\ ft$

$\Rightarrow \phi = 48.6°$

$\vec{r}_{C/B} = 2(\cos 48.6\hat{i} - \sin 48.6\hat{j})\ ft$

$\vec{V}_B = \vec{V}_A^{\,0} + \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$\vec{V}_C = \vec{V}_B + \vec{\omega}_{BC} \times \vec{r}_{C/B}$

$= \vec{\omega}_{AB} \times \vec{r}_{B/A} + \vec{\omega}_{BC} \times \vec{r}_{C/B}$

$-1\ ft/s\ \hat{i} = \omega_{AB}\hat{k} \times (3\cos 30\hat{i} + 3\sin 30\hat{j})\ ft + \omega_{BC}\hat{k} \times (2\cos 48.6\hat{i} - 2\sin 48.6\hat{j})\ ft$

$-1\ ft/s\ \hat{i} = 3\omega_{AB}\cos 30\hat{j} - 3\omega_{AB}\sin 30\hat{i} + 2\omega_{BC}\cos 48.6\hat{j} + 2\omega_{BC}\sin 48.6\hat{i}$

split into components

$\hat{i}: \quad -1 = -3\omega_{AB}\sin 30 + 2\omega_{BC}\sin 48.6$

$\hat{j}: \quad 0 = 3\omega_{AB}\cos 30 + 2\omega_{BC}\cos 48.6$

from $\hat{j}$: $-3\omega_{AB}\cos 30 = 2\omega_{BC}\cos 48.6$

$\Rightarrow \omega_{AB} = \dfrac{-2\cos 48.6}{3\cos 30}\omega_{BC} = -0.51\,\omega_{BC}$

sub into $\hat{i}$: $-1 = 3\sin 30(-0.51\,\omega_{BC}) + 2\omega_{BC}\sin 48.6$

$-1 = 0.76\,\omega_{BC} + 1.5\,\omega_{BC} = 2.26\,\omega_{BC}$

$$\Rightarrow \quad \omega_{BC} = -0.44 \text{ rad/s}$$

$$\boxed{\vec{\omega}_{BC} = -0.44 \text{ rad/s } \hat{k}}$$

from $\hat{j}$:

$$\omega_{AB} = -0.51\, \omega_{BC}$$

$$= 0.23 \text{ rad/s}$$

$$\boxed{\vec{\omega}_{AB} = 0.23 \text{ rad/s } \hat{k}}$$