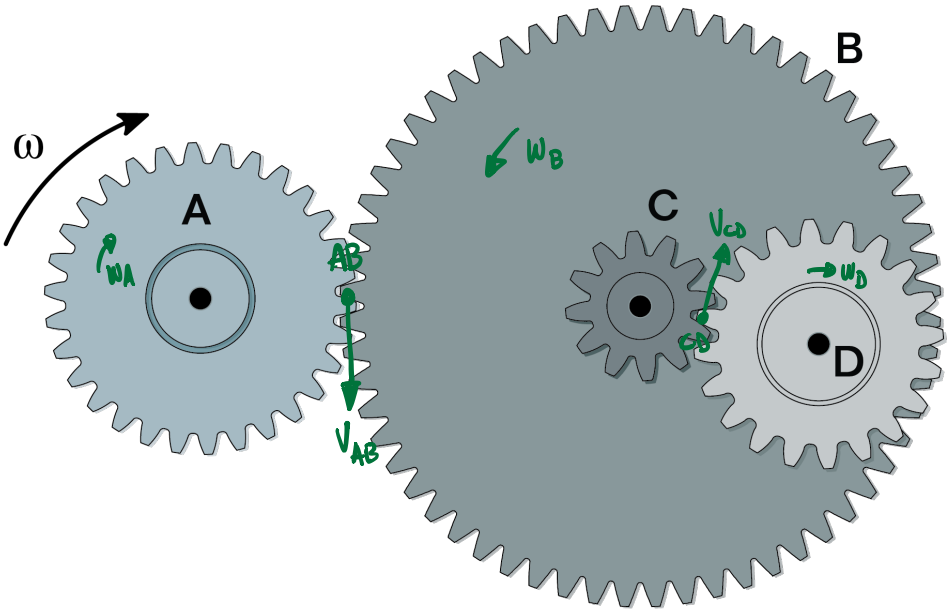
In the following gear train, gear A is in contact with gear B. Gear B and gear C are rigidly attached, and gear C is also in contact with gear D. If gear A rotates at an angular velocity of 5 rad/s in the clockwise direction, what is the magnitude of the angular velocity of gear D?

$r_A = 1\,m$

$r_B = 3\,m$

$r_C = 0.5\,m$

$r_D = 1.5\,m$



① Diagram

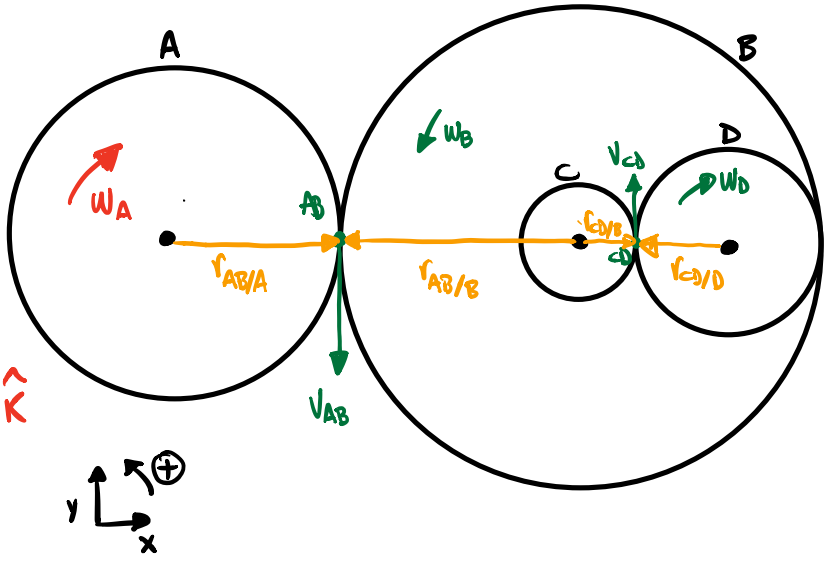
$r_A = 1\ m$

$r_B = 3\ m$

$r_C = 0.5\ m$

$r_D = 1.5\ m$

$\vec{\omega}_A = -5\ rad/s\ \hat{k}$



② Assumptions

↳ Rigid Bodies ⇒ $\omega_B = \omega_D$

③ Solution

$\vec{V} = \vec{W} \times \vec{r}$

Gear A $\Rightarrow \vec{r}_{A \to AB} = 1m\,\hat{i}$ $\qquad \vec{V}_{AB} = \vec{W}_A \times \vec{r}_{A \to AB} = -5\hat{K} \times 1\hat{i}$

$= -5\hat{j}$ m/sec

Gear B $\Rightarrow \vec{r}_{B \to AB} = -3\hat{i}$ $\qquad \vec{V}_{AB} = \vec{W}_B \times \vec{r}_{B \to AB}$

$$\begin{vmatrix} i & j & K \\ 0 & 0 & W_B \\ -3 & 0 & 0 \end{vmatrix}$$

$-5\hat{j}$ m/s $= W_B\,\hat{K} \times -3\hat{i}$

$\therefore -5\hat{j} = -3W_B\,\hat{j}$

$W_B = \frac{5}{3}$ rad/sec

★ Alternate Method $\Rightarrow \left|\frac{W_B}{W_A}\right| = \frac{r_A}{r_B} \Rightarrow |W_B| = |W_A| \frac{r_A}{r_B}$

$W_B = (5\text{ rad/sec}) \frac{(1m)}{(3m)} = \frac{5}{3}$ rad/sec

Gear C $\Rightarrow W_C = W_B \Rightarrow \left|\frac{W_D}{W_C}\right| = \frac{r_C}{r_D} \Rightarrow |W_D| = |W_C| \frac{r_C}{r_D}$

$W_D = \left|\frac{5}{3}\text{ rad/sec}\right| \frac{(0.5m)}{(1.5m)} = \frac{5}{9}$ rad/sec

$$\boxed{W_D = \frac{5}{9} \text{ rad/sec}}$$