Transcript

In this problem, we have a disk that is rotating with an angular velocity of two radians per second, and an angular acceleration of one radian per second squared in the opposite direction, we're asked to determine the velocity and acceleration vectors at point eight, which is located at a radius of one meter from the center of the disk. And at a 30 degree angle with a horizontal. First, we're going to start with the velocity. So we have our formula for the velocity, v is equal, so we have a is equal to Vg plus omega cross r. And we know that the velocity of G is zero, we can eliminate this, and we can simply solve the velocity at a is equal to omega cross r. So we know omega, because we are given omega, omega is equal to two radians per second in the k hat direction. And this is actually negative two. Because if we assume our coordinate system as follows, a positive rotation is in that direction, omega is actually pointing in the opposite direction, so we have to add a negative to that. And this is going to be crossed to our radius. And our radius is again, this vector over here. So that's starting from the point to the point of interest. And we can determine that by using with just a cosine and sine relationship. So r is going to be equal to our coasts of theta in the i hat direction, and plus r sine theta in the j hat direction. And this is simply determined from the geometry. So let me simply rearrange over here, we have our which is one meter. So one meter times cosine of 30 degrees in the i hat, plus sine of 30 degrees in the j hat direction. So we can solve this with a determinant. So we have velocity being equal to i, j, k. First we have the Omega, so the angular velocity, which is just negative two radians per second in the k hat direction. And then we are going to have our r radius, which is r, one times cosine of 30 degrees, sine of 30 degrees, and zero. Okay. And when we do this, where we saw this determinant is the eye component is going to be equal to these components times together, minus these two components times together, the J, negative j component is going to be equal to these two components multiplied together minus these two components ties together, the key component of that cross products is going to be these two multiplied together minus these two multiplied together. And since there's zeros every are in the first row, in the second row, sorry, there's two zeros. Most of these terms cancel and we're simply left with two sine of 30 degrees in the i hat direction, minus two cosine of 30 degrees in the j hat direction. And we can simply solve for the sine and cosine and sine and cosine of 30. And we get the following that velocity of A is equal to one in the i hat, minus 1.73 in the j hat, and the units are meters per second. And this is our first part of the solution. All right. The other way you can use to solve this problem is the right hand rule. So if we use, since we know that all these vectors that we're dealing with are perpendicular to each other, we can just simply use the right hand rule to solve the cross product without actually having to go through all these, the determinant. But essentially, what we want to do is we know with our right hand, we're going to point our index finger in the direction of the radius, we know that our thumb is going to point in the direction of the angular velocity, which in this case is actually into the page, because it's negative. And we see that our middle finger points in a direction that is downwards. And to the right, it's a little bit slanted, because again, we have that theta. And so what we can do is, instead of finding the cross product, we find the unit vector in the direction of the velocity and we multiply it by just the multiplication of omega times r. And in this case, we get the same exact answer. So first, we have to find the unit vector and the unit vector, again, we say the velocity is going to point in this direction, right. So this is going to be the direction of the velocity, because of the cross product with the radius, which is in this direction, and the n omega, which is going into the page. So when we do that, we find a vector that points in this direction. And if we multiply the unit vector times the total speed at point A, which we can just find by multiplying r times omega, simply simple multiplication, we get the same exact answer. So to find the velocity, the speed at a, so V, A is equal to omega times r. And this is going to be equal to two radians per second times one meter, which is two meters per second. And then we need to find the unit vector in the direction of VA. And the way we do that is we know this vector, we know this direction, and we just need to shift it by 90 degrees. So instead of pointing this way, we want it to point in that direction, so we shift it by 90 degrees. So we have our vector, our radius vector pointing in that direction. And this is the horizontal, we know theta. And we know that our unit vector in the direction of the velocity will point somehow in this direction, this angle is 90 degrees. Therefore, this angle over here with the vertical is also going to be theta. And given that, we know that, to find this unit vector u one, and we're going to add a hat because that's the sign for a unit vector, we know that u one hat is going to have the following components. So the x component is going to be the sine of theta, and I had in the positive direction, and then the y component is going to be negative cosine of theta in the j hat direction. So now that we have the unit vector, we know that the magnitude of VA times the unit vector you want, which is in the direction of VA, is going to give us the vector of VA. So we have just found the magnitude of VA, which is two meters per second. And we're going to multiply it by sine of 30 degrees in the i hat direction, minus cosine of 30 degrees in the j hat direction. And this yields one meter per second in the i hat direction minus 1.73 meters per second in the j hat direction, and this has units of meters per second. And this is our final velocity and we see that this matches with our previous results for VA. So for the acceleration, we know that a the acceleration vector is equal to alpha cross r minus omega squared R. I noticed that this omega square is not a vector. We're just taking the scalar. And we're taking a square of it, what gives the vector is this radius. So this is again, the component due to alpha. And this is again, the centripetal component due to omega. So, as you remember, the acceleration has two components, one with alpha, so the cross product, which in this case is going to point in the following direction, because alpha is in that direction, so, a Ay as ay, and this is the tangential component. And then we have one due to omega, which points in the negative r hat direction. And it's with respect to omega squared. So this is going to point in this direction here. And this is going to be the a of a acceleration at A, but this is going to be the radial component. And when we add those two vectorially, then we get the total acceleration. So let's go ahead and calculate the following. So this is going to be equal to one radian per second in the k hat direction. And again, this k hat direction is positive, because we're given it, we're given that the direction is counterclockwise, which we assumed to be a positive direction, and we're gonna cross this to that same radius. So this is going to be one meter times cosine of 30 degrees in the i hat direction, plus sine of 30 degrees in the j hat direction. And from this, then we subtract that radial component, so this is going to be minus two radians per second, all squared times that radius. And again, we already include the negative signs, our radius is still positive. So this is going to be one meter times cosine 30 degrees in the i hat direction, plus sine of 30 degrees and j hat direction. And we can go ahead and solve this, these two or this one cross product, and then add that other portion to it. So the cross product is going to be solved in the following way. So we have i, j, k. And we have 001 radian per second squared. And then at the bottom, we have our radius, which is cosine 30 degrees and sine of 30 degrees, and then zero in the Z and then we have minus the these two components over here. So minus four cosine of 30 degrees, minus four sine of 30 degrees, and then this first one is in the i hat direction. And this is in the j hat direction. So we can now go ahead and solve this the same way that I mentioned before, and get the following results. Negative sine of 30 degrees in the i hat, plus cosine of 30 degrees, j hat direction, minus four cosine of 30 degrees and the i hat, minus four sine of 30 degrees in the j hat direction. And when we pull these terms to the i, and j terms together, we get negative five sine of 30 degrees in the i hat direction, plus five cosine of 30 degrees in the j hat direction. And if we solve the sine and cosine, we get that the acceleration a is going to be equal to negative 2.5 i hat plus 4.3 j hat, meters per second squared. And this is our final answer. And again, we don't need to use this determinant method to solve this cross product, we can also use the vectors and the right hand rule. So if the same concept applies where we need to find the unit vector in the direction and then we just multiply the magnitude of the tangential acceleration in the direction of this unit vector. And this unit vector in this case, will point in that direction. So that's going to be this vector plus 90 degrees. Again, you need to watch out for these two directions, right? So alpha and omega are in opposite direction. So we're essentially just rotating by 90 degrees in the opposite direction. And you'll same processes as we did here, and you'll get to the same answer and now after so use the right hand rule just to solve this cross product here. This here, you're just keeping the same,

same method and remember to add these after the cross product to the to this and then you get the same final answer.