

Problem 20-R-WE-DK-18

This problem, you're carrying a block up an incline, and there's friction between block and the incline, you're applying a force of 10 Newtons. And you're asked to find the average power exerted during this motion. So anyone that you know that it's average power, and so it's across the whole time interval. And you're assuming that this block starts rest, and you end up with a final velocity. Okay, so the first thing we do is we start with a freebody diagram of the block. So on the block, there are, there is a friction force, there's a normal force and the friction force. And then there's the force due to gravity, and F , which pulls the block up the incline. So we're going to draw those, so we have at the center of gravity, we have f_g , then we have force F , acting on pulling up the block, so this is just F , then we have along the bottom surface, well, perpendicular to the bottom surface, we have the normal force. And then we have the force of friction, which is going to point backwards because the motion is up the incline. So f will point backwards, opposite to the motion. And then we also regarding the kinetic diagram, we have an acceleration pointing upwards, because this block is going to accelerate, okay. And everything is sloped at an angle of θ . So this angle here is θ . Okay, so now that we have our freebody diagram, we can actually calculate a sum of forces in the x and y direction. Now for simplicity, I'm going to direct my coordinate system along the incline. So x is up the incline, and y is perpendicular to that, just for simplicity, because this coordinate system is going to allow me to add these forces in just one direction, instead of splitting all these forces into two directions. If it was another coordinate system, the only course have to split up into two components is this f_g force, but it's just one compared to three other ones. Okay, so let's do a force balance in x . So the sum of forces in x is going to be equal to $m a_x$, okay, because we said we're going to have an acceleration, a_x , which in this case will just be a , so when we implement this, we got F pulling upwards, so this force here pulling along the x direction, and then we have negative f pulling backwards, so minus f , and then we have this $f_g \sin \theta$ term because this f_g on there is a $\sin \theta$ term that points backwards so minus $f_g \sin \theta$ is going to be equal to $m a$. Okay, because A is our is only in the x direction. All right, then we have our sum of forces in the y direction, which is going to be equal to zero because this there's not going to be an acceleration in the y direction, because the block is not going to detach from the surface. So we have $n - f_g \cos \theta$ is going to be equal to zero. So here we have again and pointing upwards and minus $f_g \cos \theta$ because that points downwards. Okay, so here we can directly solve for n , we have $f_g \cos \theta$, right? f_g is just mg . And θ we know so we can directly solve for and so from here, we get that n is equal to $f_g \cos \theta$, which equals to $m g \cos \theta$, which is going to be equal to one kilogram times 9.81 meters per second squared times \cos of 30 degrees. So equals to 8.496 Newton's. Okay, so now we have n and if we go to the above equation here we have, we can find f from n , because the force of friction, f is going to be equal to $\mu_k n$. And we have $f_g \sin \theta$, we're given F , so we can find the acceleration a . Okay, so let's do that. So from this equation here from one, we get that $F - \mu_k n - f_g \sin \theta = m a$ and replace f_g with mg $mg \sin \theta$ is equal to a , so we get that A is equal to $F - \mu_k n - m g \sin \theta$ over m . So A is going to be equal to 10 Newtons minus 0.2 times 8.496 Newton's minus one kilogram times 9.81. backwards, one meters per second squared, times \sin of 30 degrees over one kilogram. So we can solve for A at the a is equal to 3.39596 actually, meters per second squared, E , so this is the acceleration of the walk along x . Alright, now that we have this, we have the acceleration of the block. And we can find the time taken to travel the distance of three meters. Okay, so this acceleration is going to be constant throughout the whole system because the force is constant. And all the forces are constant throughout this whole motion. So given this acceleration, we can find the time that it takes for this block to travel the desire distance of three meters. Okay. So this is the equation that will use the distance traveled ΔS is going to be equal to the initial velocity times the time

plus one half $a t^2$, where A is the acceleration and t^2 is the time. So ΔS in our case will be three meters, because that's our desired distance of travel. v_0 is zero. So this term cancels zero, plus one half times acceleration, which we just found 3.396 meters per second squared times t^2 . Okay, so in this equation, we can directly solve for time. And if we solve this quadratic equation, we get that t is equal to 1.329 seconds. Okay, so now we have the time that it takes, and we're trying to find the time because this, we need to find we need to have a time for power. Because when we're trying to find the work or the energy expended, that is independent of time, but power is the energy over the time. So that's why we're looking at the time on because we're trying to look at power, not just the energy expended. If we had to just find energy expended, we would have to we would just multiply the force times a distance. But in this case, we need to look at time because we need to find the watts not just joules, okay. So now we have our time in which this is expended. All this energy is expended. And we can find the change in velocity of this block. And once we find the change in velocity, then we can use the change of velocity multiplied by a force, and that would be our power. Okay? So that's why we need to find this change of velocity and the change of velocity. So Δv is equal to a change in distance over change in time, right? So distance or displacement over time is velocity. So this is going to be equal to three meters divided by we are just found or time 1.329 seconds. And this is going to be equal to 2.257 meters per second. Okay, so this is the change in velocity that the block undergoes to travel this distance of three meters. Okay. And once we have this, we can quickly find power. So the power is equal to the force times the velocity. So again, to write it more formally, the change in power is $F \Delta v$. And we have Δv , and we also have our force, which is 10 Newtons. So we can equate this to 10 Newtons times 2.257 meters per second, which equals to 22.57 watts. So the power is equal to 22.57 watts. And that is our final answer. Now, there's also an alternate way of solving this question, which might be easier and more intuitive. So we know that so alternatively, we have that power is equal to some work over some time. Okay, now the work is the work that you put into the system. So in this case, it will be a force times a distance, and this is the force applied, which is 10 Newtons times the distance of three meters. So if we do this, we get 30 joules. So we expend 30 joules. And the time which this takes place is given, we've already found a 1.329 seconds, so 30 joules over 1.329 seconds, it's going to be equal to the same number 22.57 watts, which matches so this is an alternate way of finding the power