Appendix 2 Homework Problems

Problem A2.1

A shape is bounded on the left by the y axis, on the bottom by the x axis, and along its remaining side by the function $y = -\frac{1}{2}x^2 + 8$. Determine the x and y coordinates of the centroid of this shape via integration. (Hint: for $y$ bar, work from the top down to make the math easier)

Solution: $\bar{x} = 1.5$ cm, $\bar{y} = 3.2$ cm

Problem A2.2

Determine the x and y coordinates of the centroid of the shape shown below via integration.

Solution: $X_C = 2.94$ in, $Y_C = 4.24$ in
Problem A2.3

A water tank as shown below takes the form of an inverted, truncated cone. The diameter of the base is 4 ft, the diameter of the top is 8 ft, and the height of the tank is 4 ft. Using integration, determine the height of the center of mass of the filled tank. (Assume the tank is filled with water and the walls have negligible mass)

Solution: $Z_C = 2.43$ ft

Problem A2.4

Use the method of composite parts to determine the centroid of the shape shown below.

Solution: $X_C = 1.14$ in, $Y_C = 1.39$ in
Problem A2.5

A floating platform consists of a square piece of plywood weighing 50 lbs with a negligible thickness on top of a rectangular prism of a foam material weighing 100 lbs as shown below. Based on this information, what is the location of the center of mass for the floating platform?

![Diagram of floating platform]

Solution: $X_C = 3.33$ ft, $Y_C = 1.33$ ft, $Z_C = 2$ ft

Problem A2.6

Use the integration method to find the moments of inertia for the shape shown below...

i) About the x axis through the centroid.
ii) About the y axis through the centroid.

![Diagram of cone]

Solution: $I_{xx} = 6.075 \times 10^{-7}$ m$^4$, $I_{yy} = 5.0625 \times 10^{-8}$ m$^4$
A2.7

Use the integration method to find the polar moment of inertia for the semicircle shown below about point O.

![Diagram of a semicircle with r = 6 in]

Solution: \( J_{zz} = 1017.9 \text{ in}^4 \)

Problem A2.8

A plastic beam has of a four-inch square cross section with semi-circular cutouts on the top and bottom as shown below. What is the area moment of inertia of the beam’s cross section about x and y axes through the center point?

![Diagram of a beam with r = 1.5 in]

Solution: \( I_x = 7.08 \text{ in}^4, \ I_y = 17.36 \text{ in}^4 \)
Problem A2.9

A piece of angled steel has a cross section that is 1 cm thick and has a length of 6 cm on each side as shown below. What are the x and y area moments of inertia through the centroid of the cross section?

![Diagram of angled steel cross section](image1.png)

Solution: \( I_{xx} = I_{yy} = 35.462 \text{ cm}^4 = 3.546 \times 10^{-7} \text{ m}^4 \)

Problem A2.10

The pendulum in an antique clock consists of a 6 cm brass disc with a mass of .25 kg at the end of a slender wooden rod with a mass of .1 kg. Determine the mass moment of inertia of the pendulum about the top of the rod.

![Diagram of pendulum](image2.png)

(Solution: \( I_{zz} = .02026 \text{ kg m}^2 \))
Problem A2.11

A robotic leg (from the knee down) can be approximated as a slender rod with a mass of 1 kg and a length of 0.3 meters attached to the center of a flat plate with a mass of 0.75 kg measuring 0.1 x 0.05 m.

a. What is the moment of inertia of the leg about the x axis at the knee joint at the top?

b. What is the moment at the knee joint required to achieve an angular acceleration of 3 rad/s² for this leg about the x axis?

(Solution: \( I_{xx} = 0.098125 \text{ kg m}^2 \), \( M = 0.2944 \text{ Nm} \))

Problem A2.12

A space telescope can be approximated as a 600 kg cylinder with a 4-meter diameter and 4-meter height attached to two 100 kg solar panels as shown below. What is the approximate mass moment of inertia for the space telescope about the y axis shown?

(Solution: \( I_{yy} = 10,216.7 \text{ kg m}^2 \))
Problem A2.13

A flywheel has an original weight of 15 pounds and a diameter of 6 inches. To reduce the weight, four two-inch diameter holes are drilled into the flywheel, each leaving half an inch to the outside edge as shown below.

- What was the original mass moment of inertia about the center point (without holes)?
- Assuming a uniform thickness, what is the new mass moment of inertia after drilling in the holes? (Hint: holes count as negative mass in the mass moment calculations)

(Solution: $J_{\text{without holes}} = .01456 \text{ slug ft}^2$, $J_{\text{with holes}} = .01060 \text{ slug ft}^2$)